

Contents

<i>Preface</i>	vii
1 Domains, Modules and Matrices	1
1.1 Rings, Domains and Fields	1
1.2 Bezout Domains	4
1.3 \mathbb{D}_U , \mathbb{D}_P and \mathbb{D}_E Domains	9
1.4 Factorizations in $\mathbb{D}[x]$	12
1.5 Elementary Divisor Domains	16
1.6 Modules	17
1.7 Determinants	25
1.8 Algebraically Closed Fields	30
1.9 The Resultant and the Discriminant	32
1.10 The Ring $\mathbb{F}[x_1, \dots, x_n]$	37
1.11 Matrices and Homomorphisms	38
1.12 Hermite Normal Form	42
1.13 Systems of Linear Equations over Bezout Domains	51
1.14 Smith Normal Form	55
1.15 Local Analytic Functions in One Variable	60
1.16 The Local–Global Domains in \mathbb{C}^p	67
1.17 Historical Remarks	74
2 Canonical Forms for Similarity	75
2.1 Strict Equivalence of Pencils	75
2.2 Similarity of Matrices	82
2.3 The Companion Matrix	84

2.4	Splitting to Invariant Subspaces	87
2.5	An Upper Triangular Form	92
2.6	Jordan Canonical Form	96
2.7	Some Applications of Jordan Canonical Form	99
2.8	The Matrix Equation $AX - XB = 0$	103
2.9	A Criterion for Similarity of Two Matrices	109
2.10	The Matrix Equation $AX - XB = C$	114
2.11	A Case of Two Nilpotent Matrices	119
2.12	Historical Remarks	122
3	Functions of Matrices and Analytic Similarity	123
3.1	Components of a Matrix and Functions of Matrices	123
3.2	Cesaro Convergence of Matrices	127
3.3	An Iteration Scheme	131
3.4	Cauchy Integral Formula for Functions of Matrices	133
3.5	A Canonical Form over H_A	143
3.6	Analytic, Pointwise and Rational Similarity	150
3.7	A Global Splitting	154
3.8	First Variation of a Geometrically Simple Eigenvalue	155
3.9	Analytic Similarity over H_0	157
3.10	Strict Similarity of Matrix Polynomials	166
3.11	Similarity to Diagonal Matrices	169
3.12	Property L	179
3.13	Strict Similarity of Pencils and Analytic Similarity	185
3.14	Historical Remarks	192
4	Inner Product Spaces	195
4.1	Inner Product	195
4.2	Special Transformations in IPS	200
4.3	Symmetric Bilinear and Hermitian Forms	209
4.4	Max–Min Characterizations of Eigenvalues	212
4.5	Positive Definite Operators and Matrices	221

4.6	Convexity	227
4.7	Majorization	237
4.8	Spectral Functions	245
4.9	Inequalities for Traces	253
4.10	Singular Value Decomposition (SVD)	256
4.11	Characterizations of Singular Values	263
4.12	Moore–Penrose Generalized Inverse	273
4.13	Approximation by Low Rank Matrices	276
4.14	<i>CUR</i> -Approximations	281
4.15	Some Special Maximal Spectral Problems	285
4.16	Multiplicity Index of a Subspace of $\mathbf{S}(\mathbf{V})$	292
4.17	Rellich’s Theorem	300
4.18	Hermitian Pencils	304
4.19	Eigenvalues of Sum of Hermitian Matrices	316
4.20	Perturbation Formulas for Eigenvalues and Eigenvectors of Hermitian Pencils	319
4.21	Historical Remarks	323
5	Elements of Multilinear Algebra	325
5.1	Tensor Product of Two Free Modules	325
5.2	Tensor Product of Several Free Modules	333
5.3	Sparse Bases of Subspaces	344
5.4	Tensor Products of Inner Product Spaces	360
5.5	Matrix Exponents	374
5.6	Historical Remarks	392
6	Non-Negative Matrices	393
6.1	Graphs	393
6.1.1	Undirected graphs	393
6.1.2	Directed graphs	394
6.1.3	Multigraphs and multidigraphs	395
6.1.4	Matrices and graphs	399
6.2	Perron–Frobenius Theorem	403
6.3	Index of Primitivity	416
6.4	Reducible Matrices	418
6.5	Stochastic Matrices and Markov Chains	427

6.6 Friedland–Karlin Results	437
6.7 Log-Convexity	447
6.8 Min-Max Characterizations of $\rho(A)$	450
6.9 An Application to Cellular Communication	456
6.9.1 Introduction	456
6.9.2 Statement of problems	457
6.9.3 Relaxations of optimal problems	459
6.9.4 Preliminary results	460
6.9.5 Reformulation of optimal problems	464
6.9.6 Algorithms for sum rate maximization	467
6.10 Historical Remarks	472
7 Various Topics	473
7.1 Norms over Vector Spaces	473
7.2 Numerical Ranges and Radii	482
7.3 Superstable Norms	490
7.4 Operator Norms	494
7.5 Tensor Products of Convex Sets	498
7.6 The Complexity of $\text{conv } \Omega_n \odot \Omega_m$	508
7.7 Variation of Tensor Powers and Spectra	511
7.8 Variation of Permanents	521
7.9 Vivanti–Pringsheim Theorem and Applications	525
7.10 Inverse Eigenvalue Problem for Non-Negative Matrices	533
7.11 Cones	542
7.12 Historical Remarks	554
<i>Bibliography</i>	555
<i>Index of Symbols</i>	571
<i>Index</i>	579