

# C O N T E N T S

Introduction . . . . .	1
1 Vector and Matrix Algebra . . . . .	3
11 Sets and Fields . . . . .	4
111 Notion of Sets . . . . .	4
112 Composition of Sets . . . . .	5
113 Relations . . . . .	5
114 Field of Real Numbers . . . . .	6
12 Vector Algebra . . . . .	8
121 Definition of Vectors and Vector Space . . . . .	8
122 Linear Dependence and Basis of a Vector Space . . . . .	11
123 Inner Product and Euclidean Space . . . . .	14
124 Orthogonal Subspaces . . . . .	15
13 Matrices . . . . .	18
131 Matrix Definition and Matrix Composition . . . . .	18
132 Rank of a Matrix . . . . .	24
133 Computation of Inverse Matrices . . . . .	29
134 Matrix Identities . . . . .	39
135 Column Space and Null Space of a Matrix . . . . .	41
136 Determinants . . . . .	42
137 Trace of a Matrix and Vector Representation . . . . .	47
14 Quadratic Forms . . . . .	49
141 Transformations . . . . .	49
142 Eigenvalues and Eigenvectors . . . . .	52
143 Definite Matrices . . . . .	54

15	Generalized Inverses . . . . .	57
151	Right Inverse and Left Inverse . . . . .	57
152	Idempotent Matrices . . . . .	57
153	Generalized Inverse, Reflexive Generalized Inverse and Pseudoinverse . . . . .	59
154	Systems of Linear Equations . . . . .	64
155	Generalized Inverses of Symmetrical Matrices . . . . .	67
156	Properties of the Pseudoinverse and of a Special Symmetrical Reflexive Generalized Inverse . . . . .	73
16	Projections . . . . .	76
161	General Projections . . . . .	76
162	Orthogonal Projections . . . . .	77
17	Differentiation and Integration of Vectors and Matrices . . . . .	79
171	Extrema of Functions . . . . .	79
172	Derivatives of Special Functions . . . . .	82
173	Integration and Transformation of Variables . . . . .	85
2	Probability Theory . . . . .	87
21	Probability . . . . .	88
211	Introduction . . . . .	88
212	Random Events . . . . .	89
213	Axioms of Probability . . . . .	91
214	Conditional Probability and Bayes Formula . . . . .	93
215	Independent Events . . . . .	94
22	Random Variable . . . . .	95
221	Definition . . . . .	95
222	Distribution Function . . . . .	96
223	Discrete and Continuous Random Variable . . . . .	98
224	Binomial Distribution and Poisson Distribution . . . . .	100
225	Multidimensional Continuous Random Variable . . . . .	103
226	Marginal Distribution . . . . .	105
227	Conditional Distribution . . . . .	106

228	Independent Random Variables . . . . .	107
229	Transformation of Variables . . . . .	108
23	Expected Values and Moments of Random Variables . . . . .	110
231	Expectation . . . . .	110
232	Multivariate Moments . . . . .	112
233	Covariance Matrix, Error Propagation, Correlation Matrix and Weight Matrix . . . . .	115
234	Moment Generating Function . . . . .	122
24	Univariate Distributions . . . . .	125
241	Normal Distribution . . . . .	125
242	Derivation of the Normal Distribution as Distribution of Observational Errors . . . . .	128
243	Gamma Distribution . . . . .	130
244	Derivation of the Gamma Distribution as Waiting-Time Distribution . . . . .	132
245	Beta Distribution . . . . .	133
25	Multivariate Normal Distribution . . . . .	136
251	Definition and Derivation . . . . .	136
252	Moment Generating Function for the Normal Distribution . . .	137
253	Marginal Distribution and Conditional Distribution . . . . .	139
254	Independence of Normally Distributed Random Variables . . . .	141
255	Linear Functions of Normally Distributed Random Variables . .	142
256	Sum of Normally Distributed Random Variables . . . . .	143
26	Test Distributions for Univariate Models of Estimating Parameters . . . . .	144
261	$\chi^2$ -Distribution . . . . .	144
262	Non-Central $\chi^2$ -Distribution . . . . .	147
263	F-Distribution . . . . .	149
264	Non-Central F-Distribution . . . . .	151
265	t-Distribution . . . . .	154
27	Quadratic Forms . . . . .	156

271	Expected Value and Covariance . . . . .	156
272	Distribution of the Quadratic Form . . . . .	158
273	Independence of Two Quadratic Forms . . . . .	158
274	Independence of a Linear and a Quadratic Form . . . . .	159
28	Test Distributions for Multivariate Models of Estimating Parameters . . . . .	160
281	Wishart Distribution . . . . .	160
282	Derivation of the Wishart Distribution . . . . .	161
283	Distribution of the Sum of Wishart Matrices . . . . .	164
284	Distribution of the Transformed Wishart Matrix . . . . .	165
285	Distribution of Matrices of Quadratic Forms and Independence of Wishart Matrices . . . . .	166
286	Distribution of the Ratio of the Determinants of Two Wishart Matrices . . . . .	168
287	Distribution of Special Functions of Wishart Matrices . . . . .	172
3	Parameter Estimation in Linear Models . . . . .	175
31	Methods of Estimating Parameters . . . . .	177
311	Point Estimation . . . . .	177
312	Best Unbiased Estimation . . . . .	178
313	Method of Least Squares . . . . .	180
314	Maximum-Likelihood Method . . . . .	181
32	Gauss-Markoff Model . . . . .	182
321	Definition and Linearization . . . . .	182
322	Best Linear Unbiased Estimation . . . . .	185
323	Method of Least Squares . . . . .	187
324	Maximum-Likelihood Method . . . . .	190
325	Unbiased Estimation of the Variance of Unit Weight . . . . .	192
326	Numerical Computation of the Estimates and Their Covariances	195
327	Gauss-Markoff Model with Constraints . . . . .	200
328	Recursive Parameter Estimation . . . . .	207
329	Deviations from the Model . . . . .	208

33	Gauss-Markoff Model not of Full Rank. . . . .	212
331	Method of Least Squares and Maximum-Likelihood Method . . . .	212
332	Estimable Functions . . . . .	214
333	Projected Parameters as Estimable Functions . . . . .	216
334	Gauss-Markoff Model not of Full Rank with Constraints . . . .	225
34	Special Gauss-Markoff Models . . . . .	231
341	Polynomial Model . . . . .	231
342	Analysis of Variance . . . . .	234
343	Parameter Estimation for the Analysis of Variance by a Symmetrical Reflexive Generalized Inverse . . . . .	238
344	Analysis of Covariance . . . . .	242
345	Gauss-Markoff Model for Outliers in the Observations . . . .	243
35	Generalized Linear Models . . . . .	246
351	Regression Model . . . . .	246
352	Mixed Model . . . . .	249
353	Best Linear Unbiased Estimation in the Mixed Model . . . . .	251
354	Method of Least Squares and Maximum-Likelihood Method for the Mixed Model . . . . .	253
355	Model of the Adjustment with Condition Equations . . . . .	257
356	Prediction and Filtering . . . . .	258
36	Estimation of Variance and Covariance Components . . . . .	264
361	Best Invariant Quadratic Unbiased Estimation . . . . .	264
362	Locally Best Estimation . . . . .	268
363	Iterated Estimates . . . . .	272
364	Best Unbiased Estimation of the Variance of Unit Weight . . .	276
37	Multivariate Parameter Estimation . . . . .	278
371	Multivariate Gauss-Markoff Model . . . . .	278
372	Estimation of the Vectors of Parameters . . . . .	280
373	Estimation of the Covariance Matrix . . . . .	281
374	Numerical Computation of the Estimates and Incomplete Multivariate Models . . . . .	286

375	Special Model for Estimating Covariance Matrices and Estimation of Covariances for Stochastic Processes . . . . .	291
376	Multivariate Model with Constraints . . . . .	294
4	Hypothesis Testing, Interval Estimation and Test for Outliers in the Gauss-Markoff Model . . . . .	297
41	Distributions Based on Normally Distributed Observations . .	299
411	Distributions of Functions of the Residuals in the Univariate Model . . . . .	299
412	Distributions of the Covariance Matrices Estimated in the Multivariate Model . . . . .	301
42	Test of Hypotheses . . . . .	304
421	Methods for Hypothesis Testing and Likelihood Ratio Test . .	304
422	Test of a General Hypothesis in the Univariate Gauss-Markoff Model . . . . .	307
423	Special Hypotheses . . . . .	311
424	Hypothesis Testing for the Variance of Unit Weight . . . . .	316
425	Test of a General Hypothesis in the Multivariate Gauss-Markoff Model . . . . .	317
426	Hypothesis of the Identity of a Covariance Matrix with a Given Matrix . . . . .	324
43	Interval Estimation . . . . .	326
431	Confidence Intervals . . . . .	326
432	Confidence Intervals for Parameters, for Linear Functions of the Parameters and Confidence Hyperellipsoids . . . . .	327
433	Confidence Interval for the Variance of Unit Weight . . . . .	332
44	Test for Outliers . . . . .	334
441	Hypothesis Testing for Outliers . . . . .	334
442	Test for One Outlier . . . . .	336
443	Percentage Points of the Test for Outliers . . . . .	338
444	Measures of Reliability . . . . .	340

5	Discriminant Analysis . . . . .	343
51	Solution by Decision Theory . . . . .	344
511	Bayes Strategy . . . . .	344
512	Discriminant Functions . . . . .	346
52	Classification Based on the Normal Distribution . . . . .	347
521	Known Parameters . . . . .	347
522	Unknown Parameters . . . . .	348
	References . . . . .	351
	Index . . . . .	365