CONTENTS

0. INTRODUCTION 7

1. INITIAL VALUE PROBLEMS IN BANACH SPACES 9

1.1. Functions valued in Banach spaces 9
1.2. Differentiation of functions valued in Banach spaces 10
1.3. Integration of functions valued in Banach spaces 11
1.4. Properties of the integral 12
1.5. Solution of initial value problems in Banach spaces 13
1.6. Uniqueness of the solution of initial value problems 16
1.7. Initial value problems in the case of infinite systems of ordinary differential equations 17

2. SCALES OF BANACH SPACES 19

2.1. The behaviour of the derivative of a holomorphic function in compact subsets 19
2.2. Definition of scales of Banach spaces 20
2.3. Generalized Cauchy-Riemann operators in scales of Banach spaces 22
2.4. Dual scales 23

3. SOLUTION OF INITIAL VALUE PROBLEMS IN SCALES OF BANACH SPACES . 26

3.1. Differential equations in scales of Banach spaces 26
3.2. Some preliminaries 27
3.3. The method of successive approximations in scales of Banach spaces 31
3.4. Construction of sequences $\varepsilon_1, \varepsilon_2, \ldots$ and $\delta_1, \delta_2, \ldots$ 38
3.5. Existence of solutions of initial value problems in scales of Banach spaces 41
3.6. Lower bounds for the length of the convergence interval . . . 43
3.7. Uniqueness of the solution of initial value problems in scales Banach spaces 44
3.8. Linear differential equations in scales of Banach spaces . . . 48

4. THE CLASSICAL CAUCHY-KOVALEVSKAYA THEOREM 51

4.1. Statement of the problem 51
4.2. Reduction of Cauchy-Kovalevskaya systems to quasilinear first order systems 55
4.3. Proof of the classical Cauchy-Kovalevskaya theorem 59
4.4. A uniqueness theorem 65
4.5. A real variant of the classical Cauchy-Kovalevskaya theorem . 65

Bibliografische Informationen
http://d-nb.info/890884056

digitalisiert durch DEUTSCHE NATIONAL BIBLIOTHEK

5. THE HOLMGREN THEOREM 69

5.1. Statement of the problem 69
5.2. Proof of the Holmgren theorem 70
5.3. Further remarks on the classical Holmgren theorem 77
5.4. A generalization of the Holmgren theorem 81

6. BASIC PROPERTIES OF GENERALIZED ANALYTIC FUNCTIONS 84

6.1. Partial complex differentiations in the classical sense and according to Sobolev 85
6.2. Complex integral operators connected with the partial complex differentiations 89
6.3. Generalized analytic functions 99
6.4. Associated differential operators 104
6.5. Differentiability properties of associated differential operators 110

7. INITIAL VALUE PROBLEMS WITH GENERALIZED ANALYTIC INITIAL FUNCTIONS 112

7.1. Statement of the problem 112
7.2. A lemma on an overdetermined first order system 113
7.3. An inverse problem for associated differential operators . . . 117
7.4. Construction of solutions with prescribed generalized analytic initial functions 122
7.5. Uniqueness theorems for initial value problems with generalized analytic initial functions 126

8. CONTRACTION-MAPPING PRINCIPLES IN SCALES OF BANACH SPACES . . . 128

8.1. W. Walter's elementary proof of the classical Cauchy-Kovalevskaya theorem 128
8.2. Generalized analytic functions depending on time in conical domains 144
8.3. A weighted norm for functions depending on time in scales of Banach spaces 153

9. FURTHER EXISTENCE THEOREMS FOR INITIAL VALUE PROBLEMS IN SCALES OF BANACH SPACES 158

9.1. Scales of q-holomorphic and generalized q-holomorphic vectors 158
9.2. Scales of pseudoholomorphic functions in L. Bers' sense . . . 160
9.3. Commentary on connections between the Cauchy-Kovalevskaya theorem and other problems in Mathematical Analysis 161
9.4. Scales of Banach spaces in the case of more than 2 spacelike variables 165

9.5. The Ovsyannikov scale 166
9.6. Solution of initial value problems in scales of Banach spaces by Euler's polygonal line method 167
9.7. The special case of ordinary differential equations 172
9.8. Initial value problems for equations with singular coefficients 172
9.9. Cauchy-Kovalevskaya theorems for a vector-valued time variable 173

10. FURTHER UNIQUENESS THEOREMS 177

10.1. Uniqueness theorems for initial value problems in higher dimensions 177
10.2. Permanence principles 177
10.3. Uniqueness in dependence on the scale 178
10.4. A generalized Gronwall lemma for differential inequalities in scales of Banach spaces 179

REFERENCES 181

INDEX 187