

Contents

1. Symmetries in Quantum Mechanics	1
1.1 Symmetries in Classical Physics	1
1.2 Spatial Translations in Quantum Mechanics	11
1.3 The Unitary Translation Operator	12
1.4 The Equation of Motion for States Shifted in Space	13
1.5 Symmetry and Degeneracy of States	15
1.6 Time Displacements in Quantum Mechanics	19
1.7 Mathematical Supplement: Definition of a Group	21
1.8 Mathematical Supplement: Rotations and their Group Theoretical Properties	23
1.9 An Isomorphism of the Rotation Group	26
1.9.1 Infinitesimal and Finite Rotations	27
1.9.2 Isotropy of Space	29
1.10 The Rotation Operator for Many-Particle States	34
Biographical Notes	35
2. Angular Momentum Algebra Representation of Angular Momentum Operators – Generators of SO(3) –	37
2.1 Irreducible Representations of the Rotation Group	37
2.2 Matrix Representations or Angular Momentum Operators	41
2.3 Addition of Two Angular Momenta	46
2.4 Evaluation of Clebsch-Gordan Coefficients	49
2.5 Recursion Relations for Clebsch-Gordan Coefficients	50
2.6 Explicit Calculation of Clebsch-Gordan Coefficients	51
Biographical Notes	56
3. Mathematical Supplement: Fundamental Properties of Lie Groups	57
3.1 General Structure of Lie Groups	57
3.2 Interpretation of Commutators as Generalized Vector Products, Lie's Theorem, Rank of Lie Group	63
3.3 Invariant Subgroups, Simple and Semisimple Lie Groups, Ideals	65
3.4 Compact Lie Groups and Lie Algebras	69
3.5 Invariant Operators (Casimir Operators)	70
3.6 Theorem of Racah	70
3.7 Comments on Multiplets	70
3.8 Invariance Under a Symmetry Group	72
3.9 Construction of the Invariant Operators	75

3.10	Remark on Casimir Operators of Abelian Lie Groups	77
3.11	Completeness Relation for Casimir Operators	77
3.12	Review of Some Groups and Their Properties	78
3.13	The Connection Between Coordinate Transformations and Transformations of Functions	79
	Biographical Notes	87
4.	Symmetry Groups and Their Physical Meaning –	
	General Considerations	89
	Biographical Notes	93
5.	The Isospin Group (Isobaric Spin)	95
5.1	Isospin Operators for a Multi-Nucleon System	100
5.2	General Properties of Representations of a Lie Algebra	104
5.3	Regular (or Adjoint) Representation of a Lie Algebra	105
5.4	Transformation Law for Isospin Vectors	108
5.5	Experimental Test of Isospin Invariance	113
	Biographical Notes	123
6.	The Hypercharge	125
	Biographical Notes	129
7.	The SU(3) Symmetry	131
7.1	The Groups $U(n)$ and $SU(n)$	131
7.1.1	The Generators of $U(n)$ and $SU(n)$	133
7.2	The Generators of $SU(3)$	134
7.3	The Lie Algebra of $SU(3)$	136
7.4	The Subalgebras of the $SU(3)$ -Lie Algebra and the Shift Operators ...	142
7.5	Coupling of T -, U - and V -Multiplets	144
7.6	Quantitative Analysis of Our Reasoning	145
7.7	Further Remarks About the Geometric Form of an $SU(3)$ Multiplet ..	147
7.8	The Number of States on Mesh Points on Inner Shells	147
8.	Quarks and SU(3)	155
8.1	Searching for Quarks	157
8.2	The Transformation Properties of Quark States	158
8.3	Construction of all $SU(3)$ Multiplets from the Elementary Representations $[3]$ and $[\bar{3}]$	161
8.4	Construction of the Representation $D(p, q)$ from Quarks and Antiquarks	163
8.4.1	The Smallest $SU(3)$ Representations	165
8.5	Meson Multiplets	171
8.6	Rules for the Reduction of Direct Products of $SU(3)$ Multiplets	174
8.7	U -spin Invariance	176
8.8	Test of U -spin Invariance	178
8.9	The Gell-Mann-Okubo Mass Formula	180
8.10	The Clebsch-Gordan Coefficients of the $SU(3)$	182

8.11	Quark Models with Inner Degrees of Freedom	184
8.12	The Mass Formula in $SU(6)$	206
8.13	Magnetic Moments in the Quark Model	207
8.14	Excited Meson and Baryon States	209
8.15	Excited States with Orbital Angular Momentum	211
9.	Representations of the Permutation Group and Young Tableaux	213
9.1	The Permutation Group and Identical Particles	213
9.2	The Standard Form of Young Diagrams	217
9.3	Standard Form and Dimension of Irreducible Representations of the Permutation Group S_N	219
9.4	The Connection Between $SU(2)$ and S_N	225
9.5	The Irreducible Representations of $SU(n)$	227
9.6	Determination of the Dimension	232
9.7	The $SU(n - 1)$ Subgroups of $SU(n)$	235
9.8	Decomposition of the Tensor Product of Two Multiplets	237
10.	Mathematical Excursion. Group Characters	241
10.1	Definition of Group Characters	241
10.2	Schur's Lemmas	241
10.2.1	Schur's First Lemma	241
10.2.2	Schur's Second Lemma	242
10.3	Orthogonality Relations of Representations and Discrete Groups	243
10.4	Equivalence Classes	244
10.5	Orthogonality Relations of the Group Characters for Discrete Groups and Other Relations	246
10.6	Orthogonality Relations of the Group Characters for the Example of the Group D_3	247
10.7	Reduction of a Representation	248
10.8	Criterion for Irreducibility	249
10.9	Direct Product of Representations	249
10.10	Extension to Continuous, Compact Groups	250
10.11	Mathematical Excursion: Group Integration	251
10.12	Unitary Groups	252
10.13	The Transition from $U(N)$ to $SU(N)$ for the Example $SU(3)$	253
10.14	Integration over Unitary Groups	255
10.15	Group Characters of Unitary Groups	258
11.	Charm and $SU(4)$	273
11.1	Particles with Charm and the $SU(4)$	275
11.2	The Group Properties of $SU(4)$	275
11.3	Tables of the Structure Constants f_{ijk} and the Coefficients d_{ijk} for $SU(4)$	279
11.4	Multiplet Structure of $SU(4)$	280
11.5	Advanced Considerations	285
11.5.1	Decay of Mesons with Hidden Charm	285
11.5.2	Decay of Mesons with Open Charm	286

11.5.3	Baryon Multiplets	287
11.6	The Potential Model of Charmonium	295
11.7	The SU(4) [SU(8)] Mass Formula	300
11.8	The Υ Resonances	303
12.	Mathematical Supplement	307
12.1	Root Vectors and Classical Lie Algebras	307
12.2	Scalar Products of Eigenvalues	311
12.3	Cartan-Weyl Normalization	313
12.4	Graphic Representation of the Root Vectors	314
12.5	Lie Algebra of Rank 1	315
12.6	Lie Algebras of Rank 2	315
12.7	Lie Algebras of Rank $l > 2$	316
12.8	The Exceptional Lie Algebras	317
12.9	Simple Roots and Dynkin Diagrams	317
12.10	Dynkin's Prescription	319
12.11	The Cartan Matrix	321
12.12	Determination of all Roots From the Simple Roots	321
12.13	Two Simple Lie Algebras	322
12.14	Representations of the Classical Lie Algebras	323
13.	Special Discrete Symmetries	327
13.1	Space Reflection (Parity Transformation)	327
13.2	Reflected States and Operators	329
13.3	Time Reversal	330
13.4	Antiunitary Operators	331
13.5	Many-Particle Systems	335
13.6	Real Eigenfunctions	335
14.	Dynamical Symmetries	337
14.1	The Hydrogen Atom	337
14.2	The Group SO(4)	339
14.3	The Energy Levels of the Hydrogen Atom	340
14.4	The Classical Isotropic Oscillator	341
14.4.1	The Quantum Mechanical Isotropic Oscillator	342
15.	Mathematical Excursion: Non-compact Lie Groups	351
15.1	Definition and Examples of Non-compact Lie Groups	351
15.2	The Lie Group SO(2,1)	356
15.3	Application to Scattering Problems	359
Subject Index		363