## Contents

1	Excerpts from Vector and Matrix Theory
1.1	Introduction
1.2	Notations of Vectors and Vector Arithmetic
1.3	Product of Vectors
1.3.1	Inner Product of Vectors
1.3.2	Vector Product
1.4	Triple Products
1.4.1	Scalar Triple Product
1.4.2	Vector Triple Product
1.4.3	Application Examples
1.4.4	Oblique Coordinate System
1.5	Differentiation of Vectors
1.6	Matrix Notations and Simple Arithmetic of Matrices 9
1.7	Products of Matrices
1.7.1	Multiplication of a Vector and a Matrix
1.7.2	Product of Matrices
1.8	Square Matrix, Inverse Matrix and Other Related Matrices 12
1.9	Principal Directions and Eigenvalues
2	Coordinate Transformations and Displacements 15
2.1	Introduction
2.2	Coordinate Transformation Matrix 1
2.3	Calculation of Transformation Matrix
2.4	Coordinate Transformation Matrix 2
2.5	Movement and Coordinate Transformations
2.6	Application Examples
2.6.1	Successive Rotations in Space
2.6.2	Rotation of a Body Around a Line in Space
2.6.3	Calculation of Geometric Constraints
2.7	Expressions of Movement of a Body by Reflection
2.7.1	Translation
2.7.2	Rotation Around an Axis
2.7.3	Movement by Four Mirrors
2.7.4	Determination of Screw Axis, Rotation Angle and Translation Distance
2.7.5	Displacement Matrix S and Mirror Matrix M 29

•



3	Lines, Planes and Polyhedra 31
3.1	Introduction
3.2	Equations of Straight Line and Intersection of Line Segments 31
3.3	Control Polygons and Menelaus' Theorem
3.4	Equations of Plane and Intersection of Line and Plane 35
3.5	Polyhedron and Its Geometric Properties 1
3.6	Polyhedron and Its Geometric Properties 2
3.7	Interference of Polyhedra
3.8	Local Operations for Deformation of Polyhedron 42
4	Conics and Quadrics 45
4.1	Introduction
4.2	Conics
4.2.1	Equation of Conics
4.2.2	Transformation of Equation
4.2.3	Classification of Conics 47
4.2.4	Intersection of Conics 48
4.3	Quadrics
4.3.1	Coordinate Transformation
4.3.2	Classification
4.4	Intersection of Two Quadrics
5	Theory of Curves 55
5.1	Introduction
5.2	Tangent and Curvature of Curve
5.3	Binormal and Torsion of Curve
5.4	Expressions with Parameter $t$
5.5	Curvature of Space Curve and Its Projection 61
5.6	Implicit Expression of a Parametric Curve 63
6	Basic Theory of Surfaces
6.1	Introduction
6.2	The Basic Vectors and the Fundamental Magnitudes
6.3	Normal Section and Normal Curvature
6.4	Principal Curvatures
6.5	Principal Directions and Lines of Curvature
6.6	Derivatives of a Unit Normal and Rodrigues' Formula
6.7	Local Shape of Surface
7	Advanced Applications of Theory of Surfaces
7.1	Introduction
7.2	Umbilies       79

Content	s
---------	---

7.3.1	General Remarks
7.3.2	Lines of Curvature
7.3.3	Extremum Search Curves
7.3.4	Contour Curves and Their Orthogonal Curves
7.3.5	Equi-gradient Curves
7.3.6	Silhouette Curve and Silhouette Pattern
7.3.7	Highlight Curves
7.4	Characteristic Curves on a Surface 2
7.4.1	Gradient Extremum Curves or Ridge-Valley Curves
7.4.2	Loci of Zero Gaussian Curvature and Loci of Extremum Principal
1.1.4	Curvatures
7.5	Offset Surfaces
7.6	Ruled Surfaces 102
8	Curves Through Given Points, Interpolation and
	Extrapolation
8.1	Introduction
8.2	Polynomial and Rational Interpolation and Extrapolation 103
8.2.1	Lagrange's Formula
8.2.2	Numerical Methods of Interpolated Points
8.2.3	Rational Function Interpolation and Extrapolation 107
8.3	Polynomial Interpolation with Constraints of Derivatives 110
8.4	Elastic Curves with Minimum Energy
8.5	Interpolation by Parametric Curves
8.6	Appendix. Derivation of Equations by Elastic Beam Analogy 115
9	Bézier Curves and Control Points
9.1	Introduction
9.2	Curve Segment and Its Control Points
9.3	Bézier Curve and Its Operator Form
9.4	Different Expressions of <i>B</i> Curve
9.5	Derivatives at Ends of a Segment and Hodographs
9.6	Geometric Properties of B Curve
9.7	Division of a Curve Segment and Its B Polygon
9.8	Continuity Conditions of Connection of B Polygons
9.9	Elevation of Degree of a Curve Segment
9.10	Expression for a Surface Patch
9.11	Geometric Properties of a Patch
9.12	Division and Degree Elevation of a Patch
9.13	Appendix. The Original Form of the Bézier Curve

## XVII

XVIII

10	Connection of Bézier Curves and Relation to Spline
	Polygons
10.1	Introduction
10.2	Connection of B Curve Segments
10.2.1	Scale Ratios
10.2.2	Conditions of $C^{(i)}$ Connection
10.2.3	$C^{(n-1)}$ Connection and Control Points
10.2.4	Connection Defining Polygon
10.3	Introduction of S Polygon
10.3.1	Locating B Points from an S Polygon
10.3.2	Increase of Vertices of an S Polygon
10.4	B points under Geometric Connecting Condition $G^{(2)}$
10.5	Curvature Profile Problem in Design
10.5.1	Geometric Interpretation of Dividing Ratios
10.5.2	Control of Curvature Distribution of Connected Curves 158
11	Connection of Bézier Patches and Geometry of Spline
	Polygons and Nets
11.1	Introduction
11.2	Spline Nets and Connected Bézier Nets
11.2.1	Tensor Product Surfaces
11.2.2	Division of an S Net
11.3	Geometric Structure of S Polygons
11.4	Menelaus Edges and Their Dividing Points
11.4.1	Dividing Points and Sub-edges
11.4.2	Relations Among Dividing Points and Menelaus Edges 168
11.5	Derivation of B Polygons from an S Polygon
11.5.1	Reduced S Polygons
11.5.2	Examples
11.5.3	Locating B Polygons from Reduced-Truncated S Polygons 173
11.5.4	Division of an S Polygon and Insertion of a Vertex
11.6	General Formulas for Locations of $B$ Points
11.6.1	Rules of Location Symbols of $B$ Points and Their Properties 175
11.6.1.1	Level of Menelaus Edges and Dividing Points
11.6.1.2	Symbols for Location of Control Points
11.6.2	General Expressions of $B$ Point Locations $\dots \dots \dots$
11.6.2.1	Application of Location Symbols
11.6.2.2	Formulas for Location Symbols
11.6.2.3	Level of Edges of a <i>B</i> Polygon
11.7	Appendix. Orthodox Approach to a $B$ Spline Curve $\ldots$ 179
* 1	representation of the providence of the optime out to a second se

Contents

1 <b>2</b>	Rational Bézier and Spline Expressions	181
12.1	Introduction	181
12.2	Rational Bézier Curves	182
12.2.1	Rational Division Between Two Points and Its Perspective Map .	182
12.2.2	Rational Bézier Curves and Their Canonical Perspectives	184
12.2.3	Effects of Weights	187
12.2.4	Division and Degree Elevation	189
12.2.5	Derivatives at Ends of a Segment	190
12.3	Rational Bézier Patches	191
12.4	Rational Splines	
12.4.1	Rational B Polygons from a Rational S Polygon	193
12.4.2	$G^{(2)}$ Connection of Curves from a Rational S Polygon $\ldots \ldots$	196
12.5	Rational Spline Nets and Bézier Nets	197
12.6	Expressions for Conics	199
12.6.1	Conversion to an Implicit Form	199
12.6.2	Classification by Weight	201
12.6.3	Sphere and Surface of Revolution	204
12.7	Interpolation and Extrapolation with Conics	204
12.7.1	Weight of a Control Point and Parameter Values	204
12.7.2	Division of a Rational Polygon	205
12.7.3	Extension of a Curve Segment	206
12.7.4	Distance Between a Conic and a Point Near It	209
12.7.5	Curve Fitting by Conics	210
13		
	Non regular Connections of Your Sided Detabos and	
10	Non-regular Connections of Four-Sided Patches and Roundings of Corners	213
	Roundings of Corners	
13.1	Roundings of Corners	213
13.1 13.2	Roundings of Corners        Introduction        General $C^{(1)}$ Connection of B Patches	213 213
13.1 13.2 13.3	Roundings of Corners	213 213 215
13.1 13.2 13.3 13.4	Roundings of Corners	213 213 215 217
13.1 13.2 13.3 13.4 13.4.1	Roundings of Corners	213 213 215 217 217
13.1 13.2 13.3 13.4 13.4.1 13.4.2	Roundings of Corners	213 213 215 217 217 220
13.1 13.2 13.3 13.4 13.4.1 13.4.2 13.5	Roundings of Corners	213 213 215 217 217 220 224
13.1 13.2 13.3 13.4 13.4.1 13.4.2 13.5 13.6	Roundings of Corners	213 213 215 217 217 220 224 228
$13.1 \\ 13.2 \\ 13.3 \\ 13.4 \\ 13.4.1 \\ 13.4.2 \\ 13.5 \\ 13.6 \\ 13.7 \\$	Roundings of Corners	213 213 215 217 217 220 224 228 230
$13.1 \\ 13.2 \\ 13.3 \\ 13.4 \\ 13.4.1 \\ 13.4.2 \\ 13.5 \\ 13.6 \\ 13.7 \\ 13.7.1$	Roundings of Corners	213 213 215 217 217 220 224 228 230 230
$13.1 \\ 13.2 \\ 13.3 \\ 13.4 \\ 13.4.1 \\ 13.4.2 \\ 13.5 \\ 13.6 \\ 13.7 \\$	Roundings of Corners	213 213 215 217 217 220 224 228 230 230
$13.1 \\ 13.2 \\ 13.3 \\ 13.4 \\ 13.4.1 \\ 13.4.2 \\ 13.5 \\ 13.6 \\ 13.7 \\ 13.7.1$	Roundings of Corners	213 213 215 217 217 220 224 228 230 230
$13.1 \\ 13.2 \\ 13.3 \\ 13.4 \\ 13.4.1 \\ 13.4.2 \\ 13.5 \\ 13.6 \\ 13.7 \\ 13.7.1 \\ 13.7.1 \\ 13.7.2$	Roundings of Corners	213 213 215 217 220 224 228 230 230 232
13.1 13.2 13.3 13.4 13.4.1 13.4.2 13.5 13.6 13.7 13.7.1 13.7.2 14	Roundings of Corners	213 213 215 217 220 224 228 230 230 232 233
13.1 13.2 13.3 13.4 13.4.1 13.4.2 13.5 13.6 13.7 13.7.1 13.7.2 14 14.1	Roundings of Corners	213 213 215 217 220 224 228 230 230 232 233 233 233
13.1 13.2 13.3 13.4 13.4.1 13.4.2 13.5 13.6 13.7 13.7.1 13.7.2 14 14.1 14.2	Roundings of Corners	213 213 215 217 220 224 228 230 230 232 233 233 233 233 233

•

14.3.2	Two-Valued Twist Vectors and Floating Inner Control Points	237
14.4	Correction of Cross-Boundary Tangent Vectors	238
14.4.1	Connection of Four-Sided Patches	<b>238</b>
14.4.2	Evaluation and Comparison of Methods	243
14.4.3	Three-Sided Patches	
14.5	Case of $C^{(2)}$ Connection	250
14.5.1	Four-Sided Patches	250
14.5.2	Three-Sided Patches	252
15	Triangular Surface Patches and Their Connection	253
15.1	Introduction	253
15.2	Operator Form of a Triangular Patch	
15.2.1	Triangular Bézier Patches	253
15.2.2	Rational Triangular Patches	255
15.2.3	Tangents on Patch Boundaries	256
15.3	$C^{(1)}$ Connection of Triangular Patches	257
15.4	Arbitrary Connection of Three-Sided Patches	261
15.5	Division of a Triangular Patch	262
15.6	Elevation of Degree	
16	Surface Intersections	
16.1	Introduction	
16.2	Intersection of a Curved Surface and a Plane	
16.2.1	General Remarks	268
16.2.2	A Practical Method of Obtaining a Point on an Intersection	
	Curve	
16.2.3	Curve Tracing by Differential Equation Solving	270
16.3	Points on Intersection of Two Curved Surfaces	
16.3.1	General Remarks	
16.3.2	Method with an Auxiliary Plane	
16.3.3	Initial Starting Points and Critical Contact Points	275
16.3.3.1	Detection of Intersection Loops	275
16.3.3.2	Critical Points	
16.4	Intersection Curves Described by Differential Equations	282
16.4.1	Both Surfaces with Parametric Expressions	282
16.4.2	Surfaces with Implicit and Parametric Expressions	284
16.4.3	Both Surfaces with Implicit Expressions	285
16.5	Intersection Near a Probable Singular Point	
16.6	Intersection of Offset Surfaces	
16.6.1	Intersection with a Plane	290
16.6.2	Intersection of Two Offset Surfaces	
16.6.2.1	Two Parametric Surfaces	
16.6.2.2	A Parametric Surface and a Surface of Implicit Form	293

16.6.2.3	Two Surfaces with Implicit Expressions
17	Applications of the Theories in Industry 297
17.1	Introduction
17.2	Engineering Drawings and Geometric Models
17.3	Examples of Integration
17.3.1	Conventional processes
17.3.2	New Integrated Processes
17.4	Style Design System
17.4.1	Importance of Shape Design
17.4.2	Two Aspects of Style Design
17.4.3	Computer-Aided Style Design
17.4.3.1	Input of Simplified Drawings
17.4.3.2	Classes and Types of Surfaces in Style Design of Motor Cars 307
17.4.3.3	Evaluation of Curve and Surface Quality
17.4.4	Die-Face Design System
17.5	CAD/CAM of Free-Form Injection-Mold Products
Append	lix. Numerical Methods of Differential Equation Solving 323
A.1	Introduction
A.1 A.2	Introduction
A.2	Adaptive Runge-Kutta Method
A.2 A.2.1	Adaptive Runge-Kutta Method  324    Runge-Kutta Step  324
A.2 A.2.1 A.2.2	Adaptive Runge-Kutta Method324Runge-Kutta Step324Runge-Kutta with Quality Control324
A.2 A.2.1 A.2.2 A.2.3	Adaptive Runge-Kutta Method     324       Runge-Kutta Step     324       Runge-Kutta with Quality Control     324       Runge-Kutta-Fehlberg Method     325
A.2 A.2.1 A.2.2 A.2.3 A.3	Adaptive Runge-Kutta Method     324       Runge-Kutta Step     324       Runge-Kutta with Quality Control     324       Runge-Kutta-Fehlberg Method     325       Variable Stepsize Predictor-Corrector Method     326
A.2 A.2.1 A.2.2 A.2.3 A.3 A.4	Adaptive Runge-Kutta Method     324       Runge-Kutta Step     324       Runge-Kutta with Quality Control     324       Runge-Kutta-Fehlberg Method     325       Variable Stepsize Predictor-Corrector Method     326       Bulirsch-Stoer Method     327
A.2 A.2.1 A.2.2 A.2.3 A.3 A.4 A.4.1	Adaptive Runge-Kutta Method     324       Runge-Kutta Step     324       Runge-Kutta With Quality Control     324       Runge-Kutta with Quality Control     324       Runge-Kutta-Fehlberg Method     325       Variable Stepsize Predictor-Corrector Method     326       Bulirsch-Stoer Method     327       Principle of the Method     327       Outline of the Procedures     328
A.2 A.2.1 A.2.2 A.2.3 A.3 A.4 A.4.1 A.4.2	Adaptive Runge-Kutta Method     324       Runge-Kutta Step     324       Runge-Kutta with Quality Control     324       Runge-Kutta-Fehlberg Method     325       Variable Stepsize Predictor-Corrector Method     326       Bulirsch-Stoer Method     327       Principle of the Method     327
A.2 A.2.1 A.2.2 A.2.3 A.3 A.4 A.4.1 A.4.2 A.4.3	Adaptive Runge-Kutta Method324Runge-Kutta Step324Runge-Kutta with Quality Control324Runge-Kutta-Fehlberg Method325Variable Stepsize Predictor-Corrector Method326Bulirsch-Stoer Method327Principle of the Method327Outline of the Procedures328Integration Procedure328
A.2 A.2.1 A.2.2 A.2.3 A.3 A.4 A.4.1 A.4.2 A.4.3 A.4.3 A.4.4	Adaptive Runge-Kutta Method324Runge-Kutta Step324Runge-Kutta with Quality Control324Runge-Kutta with Quality Control324Runge-Kutta-Fehlberg Method325Variable Stepsize Predictor-Corrector Method326Bulirsch-Stoer Method327Principle of the Method327Outline of the Procedures328Integration Procedure328Polynomial Extrapolation329Rational Extrapolation331
A.2 A.2.1 A.2.2 A.2.3 A.3 A.4 A.4.1 A.4.2 A.4.3 A.4.4 A.4.3 A.4.4 A.4.5 A.5	Adaptive Runge-Kutta Method324Runge-Kutta Step324Runge-Kutta with Quality Control324Runge-Kutta with Quality Control324Runge-Kutta-Fehlberg Method325Variable Stepsize Predictor-Corrector Method326Bulirsch-Stoer Method327Principle of the Method327Outline of the Procedures328Integration Procedure328Polynomial Extrapolation329Rational Extrapolation331Examples and Evaluation332
A.2 A.2.1 A.2.2 A.2.3 A.3 A.4 A.4.1 A.4.2 A.4.3 A.4.4 A.4.3 A.4.4 A.4.5 A.5	Adaptive Runge-Kutta Method324Runge-Kutta Step324Runge-Kutta with Quality Control324Runge-Kutta with Quality Control324Runge-Kutta-Fehlberg Method325Variable Stepsize Predictor-Corrector Method326Bulirsch-Stoer Method327Principle of the Method327Outline of the Procedures328Integration Procedure328Polynomial Extrapolation329Rational Extrapolation331

- - - \_