

Contents

1. Introduction	1
1.1 Historical remarks	1
1.1.1 Before 1919	1
1.1.2 The period 1919–1937	3
1.1.3 The period 1963–1979	6
1.1.4 Post-1979	9
1.2 Outline of the book	11
1.3 Remarks about notation	21
2. Basic facts and the observational situation	25
2.1 The Schwarzschild lens	25
2.2 The general lens	29
2.3 The magnification factor	33
2.4 Observing gravitational lens systems	41
2.4.1 Expectations for point sources	42
2.4.2 Expectations for extended sources	46
2.5 Known gravitational lens systems	47
2.5.1 Doubles	48
2.5.2 Triples	60
2.5.3 Quadruples	64
2.5.4 Additional candidates	71
2.5.5 Arcs	72
2.5.6 Rings	77
2.5.7 A rapidly growing list of candidates	84
2.5.8 Speculations on other gravitational lens systems	84
2.5.9 Gravitational lenses and cosmology	89
3. Optics in curved spacetime	91
3.1 The vacuum Maxwell equations	91
3.2 Locally approximately plane waves	93
3.3 Fermat's principle	100
3.4 Geometry of ray bundles	104
3.4.1 Ray systems and their connection vectors	104
3.4.2 Optical scalars and their transport equations	106
3.5 Distances based on light rays. Caustics	110
3.6 Luminosity, flux and intensity	115

4.	Derivation of the lens equation	119
4.1	Einstein's gravitational field equation	119
4.2	Approximate metrics of isolated, slowly moving, non-compact matter distributions	121
4.3	Light deflection by quasistationary, isolated mass distributions	123
4.4	Summary of Friedmann–Lemaître cosmological models	127
4.5	Light propagation and redshift–distance relations in homogeneous and inhomogeneous model universes	132
4.5.1	Flux conservation and the focusing theorem	132
4.5.2	Redshift–distance relations	134
4.5.3	The Dyer–Roeder equation	137
4.6	The lens mapping in cosmology	143
4.7	Wave optics in lens theory	150
5.	Properties of the lens mapping	157
5.1	Basic equations of the lens theory	157
5.2	Magnification and critical curves	161
5.3	Time delay and Fermat's principle	166
5.4	Two general theorems about gravitational lensing	172
5.4.1	The case of a single lens plane	172
5.4.2	Generalizations	176
5.4.3	Necessary and sufficient conditions for multiple imaging	177
5.5	The topography of time delay (Fermat) surfaces	177
6.	Lensing near critical points	183
6.1	The lens mapping near ordinary images	184
6.2	Stable singularities of lens mappings	185
6.2.1	Folds. Rules for truncating Taylor expansions	186
6.2.2	Cusps	192
6.2.3	Whitney's theorem. Singularities of generic lens maps	197
6.3	Stable singularities of one-parameter families of lens mappings; metamorphoses	198
6.3.1	Umbilics	199
6.3.2	Swallowtails	203
6.3.3	Lips and beak-to-beaks	207
6.3.4	Concluding remarks about singularities	211
6.4	Magnification of extended sources near folds	215
7.	Wave optics in gravitational lensing	217
7.1	Preliminaries; magnification of ordinary images	217
7.2	Magnification near isolated caustic points	220
7.3	Magnification near fold catastrophes	222

8. Simple lens models	229
8.1 Axially symmetric lenses	230
8.1.1 General properties	230
8.1.2 The Schwarzschild lens	239
8.1.3 Disks as lenses	240
8.1.4 The singular isothermal sphere	243
8.1.5 A family of lens models for galaxies	244
8.1.6 A uniform ring	247
8.2 Lenses with perturbed symmetry (Quadrupole lenses)	249
8.2.1 The perturbed Plummer model	252
8.2.2 The perturbed Schwarzschild lens ('Chang-Refsdal lens')	255
8.3 The two point-mass lens	261
8.3.1 Two equal point masses	261
8.3.2 Two point masses with arbitrary mass ratio	264
8.3.3 Two point masses with external shear	264
8.3.4 Generalization to N point masses	265
8.4 Lenses with elliptical symmetry	266
8.4.1 Elliptical isodensity curves	267
8.4.2 Elliptical isopotentials	268
8.4.3 A practical approach to (nearly) elliptical lenses	271
8.5 Marginal lenses	274
8.6 Generic properties of "elliptical lenses"	277
8.6.1 Evolution of the caustic structure	277
8.6.2 Imaging properties	278
9. Multiple light deflection	281
9.1 The multiple lens-plane theory	282
9.1.1 The lens equation	282
9.1.2 The magnification matrix	285
9.1.3 Particular cases	287
9.2 Time delay and Fermat's principle	288
9.3 The generalized quadrupole lens	291
10. Numerical methods	295
10.1 Roots of one-dimensional equations	296
10.2 Images of extended sources	298
10.3 interactive methods for model fitting	299
10.4 Grid search methods	300
10.5 Transport of images	302
10.6 Ray shooting	303
10.7 Constructing lens and source models from resolved images	307

11. Statistical gravitational lensing:	
General considerations	309
11.1 Cross-sections	310
11.1.1 Multiple image cross-sections	311
11.1.2 Magnification cross-sections	313
11.2 The random star field	320
11.2.1 Probability distribution for the deflection	322
11.2.2 Shear and magnification	328
11.2.3 Inclusion of external shear and smooth matter density	330
11.2.4 Correlated deflection probability	334
11.2.5 <i>Spatial distribution</i> of magnifications	337
11.3 Probabilities in a clumpy universe	344
11.4 Light propagation in inhomogeneous universes	348
11.4.1 Statistics for light rays	350
11.4.2 Statistics over sources	364
11.5 Maximum probabilities	366
12. Statistical gravitational lensing: Applications	371
12.1 Amplification bias and the luminosity function of QSOs	373
12.1.1 Amplification bias: Preliminary discussion	373
12.1.2 QSO source counts and their luminosity function	378
12.2 Statistics of multiply imaged sources	380
12.2.1 Statistics for point-mass lenses	381
12.2.2 Statistics for isothermal spheres	385
12.2.3 Modifications of the lens model: Symmetric lenses	395
12.2.4 Modification of the lens model: Asymmetric lenses	399
12.2.5 Lens surveys	401
12.3 QSO-galaxy associations	404
12.3.1 Observational challenges	404
12.3.2 Mathematical formulation of the lensing problem	407
12.3.3 Maximal overdensity	408
12.3.4 Lens models	411
12.3.5 Relation to observations	415
12.4 Microlensing: Astrophysical discussion	419
12.4.1 Lens-induced variability	421
12.4.2 Microlensing in 2237 + 0305	425
12.4.3 Microlensing and broad emission lines of QSOs ..	429
12.4.4 Microlensing and the classification of AGNs	433

12.5	The amplification bias: Detailed discussion	435
12.5.1	Theoretical analysis	435
12.5.2	Observational hints of amplification bias	444
12.5.3	QSO-galaxy associations revisited	447
12.6	Distortion of images	448
12.7	Lensing of supernovae	453
12.8	Further applications of statistical lensing	456
12.8.1	Gravitational microlensing by the galactic halo ..	456
12.8.2	Recurrence of γ -ray bursters	460
12.8.3	Multiple imaging from an ensemble of galaxies, and the 'missing lens' problem	461
13.	Gravitational lenses as astrophysical tools	467
13.1	Estimation of model parameters	468
13.1.1	Invariance transformations	471
13.1.2	Determination of lens mass and Hubble constant	473
13.1.3	Application to the 0957 + 561 system	476
13.2	Arcs in clusters of galaxies	483
13.2.1	Introduction	483
13.2.2	The nearly spherical lens	485
13.2.3	Analysis of the observations; arcs as astronomical tools	492
13.2.4	Statistics of arcs and arclets	498
13.3	Additional applications	501
13.3.1	The size of QSO absorption line systems	501
13.3.2	Scanning of the source by caustics	504
13.3.3	The parallax effect	508
13.3.4	Cosmic strings	509
13.3.5	Upper limits to the mass of some QSOs	511
13.3.6	Gravitational lensing and superluminal motion ..	512
13.4	Miscellaneous topics	513
13.4.1	Lensing and the microwave background	513
13.4.2	Light deflection in the Solar System	514
13.4.3	Light deflection in strong fields	514
References	517
Index of Individual Objects	545
Subject Index	547