

# Contents

## Part I: VSS and SM Algorithms and Their Analysis

<b>1</b>	<b>Sliding Mode Enforcement after 1990: Main Results and Some Open Problems</b> .....	<b>3</b>
	L. Fridman	
1.1	Sliding Mode Control Up until 1990 .....	3
1.2	Second Order Sliding Modes: First Generation .....	4
1.2.1	Twisting Algorithm .....	4
1.2.2	The First Criticism of SOSM .....	5
1.2.3	The Super-Twisting Algorithm .....	5
1.2.4	The Sub-Optimal Algorithm .....	7
1.2.5	Recapitulations .....	7
1.3	Second Generation: Arbitrary Order Sliding Mode Controllers .....	8
1.3.1	Discussion on the Definition of $r$ -th Order Sliding Motions .....	8
1.3.2	Arbitrary Order Sliding Mode Controllers .....	9
1.3.3	Black-Box Control .....	11
1.4	Terminal Sliding Mode Control .....	12
1.5	Third Generation: Non-Homogeneous HOSMs .....	15
1.6	Lyapunov Based Approach .....	15
1.7	Chattering Problem and HOSM .....	16
1.7.1	Chattering Analysis in the Frequency Domain .....	17
1.7.2	Singularly Perturbed Analysis of Homogeneous Sliding Modes in the Presence of Fast Actuators .....	22
1.7.3	Energy Based Approach .....	25
1.7.4	Recapitulation .....	26
1.7.5	Open Problems .....	26
1.8	HOSM Observation and Identification .....	26
1.8.1	HOSM Observation and Unknown Inputs Identification .....	27

- 1.8.2 Time Invariant Parameter Identification ..... 30
- 1.8.3 Further Development ..... 32
- 1.8.4 Recapitulation ..... 32
- 1.8.5 Open Problems ..... 33
- 1.9 Integral Sliding Mode Control ..... 33
- 1.10 HOSM Output Based Control ..... 35
  - 1.10.1 Open Problems ..... 37
- 1.11 Adaptive Sliding Mode Control ..... 37
  - 1.11.1 ASMC with Known Bounds for the Disturbance ..... 37
  - 1.11.2 ASMC without Known Bounds for the Disturbance .... 38
- 1.12 HOSM Based Unmatched Uncertainties Compensation..... 38
  - 1.12.1 Black Box Control via HOSM ..... 39
  - 1.12.2 Model Based Application of HOSM ..... 39
  - 1.12.3 Exact Unmatched Uncertainties Compensation  
Based on HOSM Observation ..... 40
  - 1.12.4 Conclusions ..... 41
- 1.13 VSS Methods for Hybrid Systems ..... 42
  - 1.13.1 Hybrid Nonlinear Systems ..... 42
  - 1.13.2 Hybrid Linear Systems ..... 42
  - 1.13.3 Open Problems ..... 42
- 1.14 Relay Control with Delay ..... 43
  - 1.14.1 Oscillatory Nature of Relay Delayed Systems ..... 43
  - 1.14.2 Methods of the Relay Delayed Control Design ..... 45
  - 1.14.3 Prediction Method ..... 45
  - 1.14.4 Methods of Oscillation Control ..... 46
  - 1.14.5 Open Problems ..... 47
- 1.15 Distributed Parameter Systems ..... 47
- References ..... 49

- 2 Finite-Time Stability and High Relative Degrees in Sliding-Mode Control ..... 59**
  - Arie Levant
  - 2.1 Introduction ..... 59
  - 2.2 Preliminaries ..... 62
  - 2.3 SISO Regulation Problem ..... 63
    - 2.3.1 Standard SISO Regulation Problem and the Idea of  
Its Solution ..... 63
  - 2.4 Homogeneity, Finite-Time Stability and Accuracy ..... 64
  - 2.5 Homogeneous Sliding Modes ..... 67
    - 2.5.1 Second Order Sliding Mode Controllers ..... 67
    - 2.5.2 Arbitrary Order Sliding Mode Controllers ..... 69
  - 2.6 Differentiation and Output-Feedback Control ..... 72
    - 2.6.1 Arbitrary Order Robust Exact Differentiation ..... 72
    - 2.6.2 Output-Feedback Control ..... 73
  - 2.7 Adjustment of the Controllers ..... 75

2.7.1	Control Magnitude Adjustment .....	75
2.7.2	Parameter Adjustment .....	76
2.8	Advanced Issues .....	77
2.8.1	Chattering Analysis .....	77
2.8.2	Robustness Issues .....	81
2.8.3	Choosing the Parameters .....	82
2.9	Application and Simulation Examples .....	82
2.9.1	Control Simulation .....	82
2.9.2	Signal Processing: Real-Time Differentiation .....	86
2.9.3	Image Processing .....	88
2.10	Conclusions .....	90
	References .....	90
<b>3</b>	<b>Frequency-Domain Methods in Conventional and Higher-Order Sliding Mode Control</b> .....	<b>93</b>
	Igor M. Boiko	
3.1	Introduction .....	93
3.2	Ideal and Real Sliding Modes: Poincare Maps and Frequency-Domain Approach .....	94
3.3	Analysis of Convergence – Quasi-Static DF Approach .....	96
3.4	Frequency-Domain Characteristics and Convergence Rate .....	102
3.5	Extension to Higher-Order Plants .....	106
3.6	Extention to Other Types of Controllers .....	109
3.7	Conclusions .....	111
	References .....	112
<b>4</b>	<b>Lyapunov Approach for Analysis and Design of Second Order Sliding Mode Algorithms</b> .....	<b>113</b>
	Jaime A. Moreno	
4.1	Introduction .....	113
4.2	Problem Statement and Main Results .....	116
4.3	Finite Time Convergence of Unperturbed GSOA with Constant Gains .....	119
4.3.1	Stability Analysis without Perturbations: An ALE Approach .....	119
4.3.2	Convergence Time .....	120
4.4	Robustness and Exactness of the Perturbed GSOA with Constant Gains .....	121
4.4.1	The Class of Perturbations .....	121
4.4.2	Robust Stability Analysis: A Riccati Inequality Approach .....	122
4.4.3	Practical Stability .....	123
4.4.4	Frequency Domain Interpretation: The Circle Criterium .....	124

4.5	Uniformity of the GSOA with Constant Gains .....	126
4.5.1	A Non Quadratic Strong Lyapunov Function for the GSOA .....	126
4.5.2	Uniformity in the Convergence .....	127
4.5.3	An Alternative Robust Lyapunov Function .....	128
4.6	The GSOA with Variable Gains .....	128
4.7	Conclusions .....	129
4.8	Appendix .....	130
4.8.1	Proof of Theorem 4.1 .....	132
4.8.2	Proof of Proposition 4.1 .....	134
4.8.3	Proof of Theorem 4.2 .....	135
4.8.4	Proof of Theorem 4.3 .....	135
4.8.5	Proof of Theorem 4.4 .....	138
4.8.6	Proof of Proposition 4.2 .....	141
4.8.7	Proof of Proposition 4.3 .....	143
4.8.8	Proof of Theorem 4.5 .....	146
	References .....	147
<b>5</b>	<b>A New Design of Sliding Mode Control Systems .....</b>	<b>151</b>
	Zhihong Man, Suiyang Khoo, Xinghuo Yu, Chunyan Miao, Jiong Jin, Feisiang Tay	
5.1	Introduction .....	152
5.2	Problem Formulation .....	154
5.3	Convergence Analysis .....	157
5.4	A Simulation Example .....	161
5.5	Conclusions .....	165
	References .....	166
<b>6</b>	<b>Second-Order Sliding Mode Approaches to Control and Estimation for Fractional Order Dynamics .....</b>	<b>169</b>
	A. Pisano, M. Rapačić, E. Usai	
6.1	Introduction .....	169
6.2	Preliminaries on Fractional Calculus .....	171
6.3	Second-Order Sliding Mode Controllers for Multivariable Linear FOS .....	173
6.3.1	Sliding Manifold Design .....	174
6.3.2	Control-Input Design .....	177
6.3.3	Implementation Issues .....	178
6.3.4	Simulation Results .....	180
6.4	Second-Order Sliding Mode Based Observation and Estimation for FOS .....	181
6.4.1	Disturbance Observer for FOS .....	181
6.4.2	Discrete-Mode Identification for Switched FOS .....	184
6.4.3	Simulation Results .....	189
6.4.4	Disturbance Estimation Test .....	189

6.4.5	Discrete State Estimation Test .....	190
6.5	Experimental Fault Detection of a Hydraulic Plant .....	192
6.6	Conclusions .....	194
	References .....	194
<b>7</b>	<b>Discussion about Sliding Mode Algorithms, Zeno Phenomena and Observability .....</b>	<b>199</b>
	L. Yu, J.-P. Barbot, D. Benmerzouk, D. Boutat, T. Floquet, G. Zheng	
7.1	Discussion on Zeno and Sliding Mode Behavior .....	200
7.2	Zeno Types .....	201
7.3	Mathematical Recalls of H-K Integral .....	202
7.4	Observability and Observer Design for Some Classes of Hybrid Dynamical System .....	205
7.4.1	First Basic Observability Form .....	205
7.4.2	Second Basic Observability Form .....	205
7.4.3	Extended Observability Form .....	205
7.4.4	Discussion on the Observability of the First Basic Observability Form .....	206
7.4.5	Discussion on the Observability of the Second Basic Observability Form .....	206
7.4.6	Observability for the Extended Observability Form ....	209
7.5	The Two Tanks Example .....	211
7.6	Conclusion .....	217
	References .....	217
 <b>Part II: Sliding Mode Control Design</b>		
<b>8</b>	<b>Output Feedback Sliding Mode Control of Uncertain Systems in the Presence of State Delay with Applications .....</b>	<b>223</b>
	X. Han, E. Fridman, S.K. Spurgeon	
8.1	Introduction .....	223
8.2	Problem Formulation .....	226
8.3	A General Framework for Design .....	226
8.4	Existence Problem .....	227
8.5	Reachability Problem .....	232
8.6	Liquid Monopropellant Rocket Motor Control .....	238
8.7	Conclusion .....	242
	References .....	242
<b>9</b>	<b>Sliding Mode Controller Design: An Input-Output Approach .....</b>	<b>245</b>
	Hebertt Sira-Ramírez, Alberto Luviano-Juárez, John Cortés-Romero	
9.1	Introduction .....	245
9.2	An Introductory Example .....	247
9.2.1	An Average GPI Controller Design Devoid of Observers .....	247

9.2.2	A Switched Control Interpretation of the Average Design.....	249
9.2.3	A GPI Observer Based Approach .....	250
9.3	Definitions and Main Results .....	254
9.4	An Application Example with Simulations .....	257
9.4.1	An Observer-Free Approach .....	258
9.4.2	A GPI Observer Based Approach .....	259
9.5	A “Buck” Converter Example with Experimental Results .....	261
9.5.1	The Buck Converter Model .....	262
9.5.2	The GPI Sliding Mode Controller .....	263
9.5.3	Experimental Results .....	264
9.6	Conclusions .....	267
	References .....	268
<b>10</b>	<b>Output Feedback Sliding Mode Control Approaches Using Observers and/or Differentiators.....</b>	<b>269</b>
	Liu Hsu, Eduardo V.L. Nunes, Tiago Roux Oliveira, Alessandro Jacoud Peixoto, José Paulo V.S. Cunha, Ramon R. Costa, Fernando Lizarralde	
10.1	Introduction .....	270
10.2	Preliminaries .....	271
10.3	Problem Statement .....	271
10.3.1	Basic Assumptions .....	272
10.3.2	Control Objective .....	273
10.4	Output Tracking Error Equation .....	273
10.4.1	Output Feedback Model Matching Control .....	273
10.4.2	Error Equation and Equivalent Nonlinear Input Disturbance .....	274
10.5	Norm State Observer and Norm Bound for Equivalent Disturbance .....	274
10.6	Output Feedback Sliding Mode Controller .....	275
10.7	Relative Degree Compensation .....	276
10.7.1	Linear Lead Filter .....	276
10.7.2	Variable Structure Lead Filter .....	276
10.7.3	High-Gain Observers .....	278
10.7.4	Hybrid Estimation Scheme.....	282
10.8	Peaking Phenomena Avoidance .....	284
10.9	Chattering Alleviation .....	285
10.10	Binary Model Reference Adaptive Control .....	286
10.11	Experimental Results .....	287
10.12	Concluding Remarks .....	289
	References .....	290

<b>11</b>	<b>Sliding Modes for Fault Detection and Fault Tolerant Control</b> . . . . .	293
	C. Edwards, H. Alwi, C.P. Tan, J.M. Andrade da Silva	
11.1	Introduction . . . . .	294
11.2	Sliding Mode Observers for Fault Detection . . . . .	295
11.3	A Cascade Based Robust Fault Reconstruction Scheme . . . . .	297
	11.3.1 Summary of Fault Reconstruction Algorithm . . . . .	298
	11.3.2 Design Example . . . . .	303
11.4	Reconstruction of Incipient Sensor Faults . . . . .	305
	11.4.1 Simulation Results . . . . .	310
11.5	Unmatched Parametric Uncertainty . . . . .	311
11.6	Fault Tolerant Control . . . . .	313
	11.6.1 Design Procedures . . . . .	314
	11.6.2 Benchmark Simulation Results . . . . .	317
11.7	Conclusions . . . . .	320
	References . . . . .	321
<b>12</b>	<b>Applying Sliding Mode Technique to Optimal Filter and Controller Design</b> . . . . .	325
	Michael Basin	
12.1	Introduction . . . . .	326
12.2	Optimal Filtering Problem . . . . .	327
	12.2.1 Problem Statement . . . . .	327
12.3	Mean-Square Filter Design . . . . .	328
	12.3.1 Example 1 . . . . .	329
	12.3.2 Appendix 1 . . . . .	330
12.4	Mean-Module Filter Design . . . . .	332
	12.4.1 Example 2 . . . . .	332
	12.4.2 Appendix 2 . . . . .	333
12.5	Optimal Controller Problem . . . . .	335
	12.5.1 Problem Statement . . . . .	335
12.6	Mean-Square Controller Design . . . . .	337
	12.6.1 Separation Principle. I . . . . .	337
	12.6.2 Optimal Controller Problem Solution. I . . . . .	339
	12.6.3 Example 3 . . . . .	340
12.7	Mean-Module Controller Design . . . . .	342
	12.7.1 Separation Principle. II . . . . .	342
	12.7.2 Optimal Controller Problem Solution. II . . . . .	343
	12.7.3 Example 4 . . . . .	344
12.8	Conclusions . . . . .	347
	References . . . . .	348
<b>13</b>	<b>Output Tracking and Observation in Nonminimum Phase Systems via Classical and Higher Order Sliding Modes</b> . . . . .	351
	Y. Shtessel, S. Baev, C. Edwards, S. Spurgeon, A. Zinober	
13.1	Introduction . . . . .	352

13.2	System Description .....	353
13.3	Problem Formulation .....	355
13.4	Sliding Mode Control Design .....	355
13.4.1	The Extended Method of Stable System Center .....	355
13.4.2	Sliding Variable and Sliding Mode Control .....	361
13.4.3	The Nonminimum Phase System Output Tracking Error Dynamics in the Sliding Mode .....	362
13.5	Observer for the Unstable Internal Dynamics .....	363
13.5.1	Luenberger Observer Design .....	363
13.5.2	Reconstruction of the Internal State .....	364
13.5.3	Reconstruction of the External Disturbance .....	364
13.6	Case Study 1: Output Voltage Tracking in Nonminimum Phase DC/DC Electric Power Converter .....	364
13.6.1	Model of the Boost DC/DC Converter .....	365
13.6.2	The Problem Formulation .....	367
13.6.3	Sliding Mode Controller Design .....	367
13.6.4	Generation of a Bounded Profile $\eta_c$ .....	368
13.6.5	Sliding Mode Parameter Observer .....	368
13.6.6	Numerical Simulations .....	369
13.7	Case Study 2: SISO Output Tracking in Systems with Time Delay in Control Feedback .....	371
13.7.1	Preliminaries .....	372
13.7.2	Problem Formulation .....	373
13.7.3	Padé Approximation .....	373
13.7.4	Design of a Sliding Mode Controller for Causal Output Tracking .....	374
13.7.5	Numerical Example .....	376
13.8	Conclusions .....	378
	References .....	378
<b>14</b>	<b>Discrete-Time Sliding Mode Control Using Output Feedback and Nonlinear Surface .....</b>	<b>381</b>
	Bijnan Bandyopadhyay, Fulwani Deepak	
14.1	Introduction .....	381
14.2	Multirate Output Feedback .....	384
14.3	Nonlinear Sliding Surface .....	386
14.4	Control Laws .....	390
14.4.1	Control Law Based on Reaching Law Approach .....	391
14.4.2	Control Law with Disturbance Observer .....	393
14.5	Extension to Input-Delay Systems .....	394
14.6	Magnetic Tape Position Tracking .....	395
14.6.1	Comparison with Different Linear Sliding Surfaces ....	400
14.6.2	Nonlinear Sliding Surface with Disturbance .....	401
14.7	Summary .....	403
	References .....	403



**Part III: Applications**

<b>15</b>	<b>Higher Order Sliding Modes in Collaborative Robotics</b> .....	409
	Michael Defoort, Thierry Floquet, Anne-Marie Kőkösy, Wilfrid Perruquetti	
15.1	Introduction .....	409
15.2	Some Contributions on Higher Order Sliding Mode .....	412
15.2.1	Problem Formulation .....	412
15.2.2	Design of a Higher Order Sliding Mode Controller .....	414
15.3	Collaborative Robotics Issues .....	417
15.3.1	Context .....	417
15.3.2	Control Issues .....	417
15.3.3	Problem Formulation and Navigation Strategy .....	418
15.3.4	Path Planning .....	421
15.3.5	Path Tracking .....	426
15.3.6	Experimental Results .....	431
15.4	Conclusion .....	434
	References .....	435
<b>16</b>	<b>Two Applications of Sliding Mode Control in Energy Generation and Power Electronics</b> .....	439
	D. Biel, A. Dòria-Cerezo, E. Fossas, R.S. Muñoz-Aguilar, R. Ramos-Lara	
16.1	Introduction .....	439
16.2	Sliding Mode Control of a Wound Rotor Synchronous Generator .....	440
16.2.1	System Description .....	441
16.2.2	Direct Sliding Mode Controller .....	442
16.2.3	Sliding Mode Control with an Outer-PI Loop .....	445
16.2.4	Dynamic Sliding Mode Controller .....	446
16.2.5	Simulations .....	447
16.2.6	Experimental Results .....	449
16.3	Implementing ON/OFF Controllers by Field Parallel Gate Arrays (FPGA) .....	451
16.3.1	Chattering Reduction .....	452
16.3.2	A $m$ -Phases Parallel Buck Converter .....	455
16.4	Conclusions .....	457
	References .....	463
<b>17</b>	<b>Advances in High Order and Adaptive Sliding Mode Control – Theory and Applications</b> .....	465
	F. Plestan, V. Brégeault, A. Glumineau, Y. Shtessel, E. Moulay	
17.1	Introduction .....	465
17.2	Problem Statement .....	466
17.3	Adaptive Sliding Mode Control .....	469

17.4	High Order Sliding Mode Control	472
17.4.1	A Unified Approach	472
17.4.2	Control Solutions and Design of Functions $\mathcal{F}$	474
17.4.3	Second Order Sliding Mode Control by Static Output Feedback	478
17.5	Applications	480
17.5.1	Control of Electropneumatic Actuator	480
17.5.2	Control of Induction Motor	486
17.6	Conclusion	490
	References	491
<b>18</b>	<b>Sliding Mode Controllers and Observers for Electromechanical Systems</b>	<b>493</b>
	J. de Leon-Morales	
18.1	Introduction	493
18.1.1	Application Domains of Sliding Mode	494
18.1.2	Paper Structure	495
18.2	Power Systems: Synchronous Machine and Multi-machine Systems	495
18.2.1	Synchronous Machine	495
18.2.2	One Axes Model	497
18.2.3	Sliding-Mode Controller Design	498
18.2.4	Multi-machine Mathematical Model	501
18.3	Helicopter: Twin Rotor System	506
18.3.1	Dynamical Model of a Twin Rotor System	506
18.4	Teleoperation Bilateral: Master-Slave Systems	510
18.4.1	Introduction	510
18.4.2	Teleoperation System	510
18.4.3	Controller Design	512
18.4.4	Super Twisting Observer Design	513
18.4.5	Simulation Results	514
18.5	Conclusions	515
	References	515
<b>19</b>	<b>Synthesis of Canonical Elements for Power Processing Based on Sliding-Mode Control</b>	<b>517</b>
	Luis Martínez-Salamero and Angel Cid-Pastor	
19.1	Introduction	517
19.2	Power Processing Systems	519
19.3	Generalized Canonical Element	520
19.4	Synthesis of DC Transformers	521
19.5	Power Gyration	527
19.6	Loss-Free Resistors	528
19.7	Impedance Matching	529
19.8	DC-AC Conversion	534

19.9	Power Distribution .....	536
19.10	Power Factor Correction .....	537
19.11	Conclusions .....	538
	References .....	539
<b>20</b>	<b>Second Order Sliding Modes to Control and Supervise Industrial Robot Manipulators</b> .....	<b>541</b>
	Antonella Ferrara, Luca Massimiliano Capisani	
20.1	Introduction .....	541
20.2	Problem Formulation .....	542
20.2.1	The Manipulator Model .....	543
20.3	Solution to the Problem 1: Robust Motion Control for Robot Manipulators .....	543
20.3.1	Design of the Inverse Dynamics Part of the Control Scheme .....	544
20.3.2	Design of the Proposed Second Order Sliding Mode Controller .....	544
20.4	Experimental Results on Motion Control .....	546
20.4.1	The Considered Industrial Robot .....	547
20.4.2	The Experiments .....	548
20.4.3	Comparison with the Super Twisting Second Order Sliding Mode Algorithm .....	553
20.5	Solution to Problem 2: Fault Diagnosis for Robot Manipulators .....	555
20.5.1	The Considered Fault Scenarios .....	555
20.5.2	Actuator and Sensor Faults .....	555
20.5.3	The Proposed Diagnostic Scheme .....	555
20.5.4	Actuator Faults Detection Strategy .....	556
20.5.5	Sensor Faults Detection Strategy .....	557
20.5.6	Residual Generation .....	559
20.5.7	Fault Isolation for Single Faults .....	560
20.5.8	Experimental Results on Fault Diagnosis .....	561
20.5.9	Experimental Test in Presence of Actuator Faults .....	561
20.5.10	Experimental Tests in Presence of Sensor Faults .....	562
20.6	Conclusions .....	563
	References .....	564
<b>21</b>	<b>Sliding Block Control of Electrical Machines (Motors and Generators)</b> .....	<b>569</b>
	Alexander G. Loukianov, Jose M. Cañedo, B. Castillo-Toledo, Edgar N. Sanchez	
21.1	Introduction .....	569
21.2	Synchronous Motor SM Block Control .....	570
21.2.1	Plant Model .....	570
21.2.2	The Flux Linkage $\psi_{fd}$ Control Loop .....	571

21.2.3	The Current $i_d$ Control Loop .....	572
21.2.4	The Speed $\omega_m$ Control Loop .....	572
21.3	The Synchronous Generator Control .....	573
21.3.1	Plant Model .....	573
21.3.2	The Idea of Block Integral SM Controllers .....	574
21.3.3	Block Integral SM Speed Stabilizer .....	576
21.3.4	SM Voltage Regulator .....	578
21.3.5	Control Switching Logic .....	579
21.4	Induction Motor with Magnetic Saturation Control .....	579
21.4.1	Mathematical Model of Induction Motor with Saturation .....	580
21.5	Induction Motor Discrete Time Control .....	581
21.5.1	Plant Model .....	581
21.5.2	Control Algorithm .....	584
21.6	Induction Motor Hybrid Control .....	588
21.6.1	Discrete-Time Controller .....	589
21.7	Induction Motor Neural Network SM Block Control .....	591
21.7.1	Identification .....	591
21.7.2	Controller Design .....	593
	References .....	594