

# Contents

<b>1 Graph Theory and Small-World Networks</b> .....	1
1.1 Graph Theory and Real-World Networks .....	1
1.1.1 The Small-World Effect .....	1
1.1.2 Basic Graph-Theoretical Concepts .....	3
1.1.3 Graph Spectra and Degree Distributions .....	8
1.2 Percolation in Generalized Random Graphs .....	14
1.2.1 Graphs with Arbitrary Degree Distributions .....	15
1.2.2 Probability Generating Function Formalism .....	20
1.2.3 Distribution of Component Sizes .....	22
1.3 Robustness of Random Networks .....	26
1.4 Small-World Models .....	30
1.5 Scale-Free Graphs .....	32
Exercises .....	37
Further Reading .....	39
References .....	39
<b>2 Chaos, Bifurcations and Diffusion</b> .....	41
2.1 Basic Concepts of Dynamical Systems Theory .....	41
2.2 Bifurcations and Stability .....	47
2.3 The Logistic Map and Deterministic Chaos .....	51
2.4 Dissipation and Adaption .....	56
2.4.1 Dissipative Systems and Strange Attractors .....	57
2.4.2 Adaptive Systems .....	61
2.5 Diffusion and Transport .....	66
2.5.1 Random Walks, Diffusion and Lévy Flights .....	66
2.5.2 The Langevin Equation and Diffusion .....	69
2.6 Noise-Controlled Dynamics .....	71
2.6.1 Stochastic Escape .....	72
2.6.2 Stochastic Resonance .....	75
2.7 Dynamical Systems with Time Delays .....	78

Exercises .....	81
Further Reading .....	82
References .....	83
<b>3 Complexity and Information Theory .....</b>	<b>85</b>
3.1 Probability Distribution Functions .....	85
3.1.1 Bayesian Statistics .....	88
3.1.2 The Law of Large Numbers .....	91
3.1.3 Time Series Characterization .....	92
3.2 Entropy and Information .....	96
3.2.1 Information Content of a Real-World Time Series .....	102
3.2.2 Mutual Information .....	103
3.2.3 Kullback-Leibler Divergence .....	108
3.3 Complexity Measures .....	110
3.3.1 Complexity and Predictability .....	112
3.3.2 Algorithmic and Generative Complexity .....	114
Exercises .....	116
Further Reading .....	118
References .....	118
<b>4 Random Boolean Networks .....</b>	<b>121</b>
4.1 Introduction .....	121
4.2 Random Variables and Networks .....	123
4.2.1 Boolean Variables and Graph Topologies .....	123
4.2.2 Coupling Functions .....	125
4.2.3 Dynamics .....	127
4.3 The Dynamics of Boolean Networks .....	128
4.3.1 The Flow of Information Through the Network .....	129
4.3.2 The Mean-Field Phase Diagram .....	131
4.3.3 The Bifurcation Phase Diagram .....	133
4.3.4 Scale-Free Boolean Networks .....	137
4.4 Cycles and Attractors .....	139
4.4.1 Quenched Boolean Dynamics .....	139
4.4.2 The $K = 1$ Kauffman Network .....	142
4.4.3 The $K = 2$ Kauffman Network .....	144
4.4.4 The $K = N$ Kauffman Network .....	145
4.5 Applications .....	147
4.5.1 Living at the Edge of Chaos .....	147
4.5.2 The Yeast Cell Cycle .....	149
4.5.3 Application to Neural Networks .....	151
Exercises .....	153
Further Reading .....	154
References .....	154

<b>5 Cellular Automata and Self-Organized Criticality</b> .....	157
5.1 The Landau Theory of Phase Transitions .....	157
5.2 Criticality in Dynamical Systems .....	162
5.2.1 1/f Noise.....	166
5.3 Cellular Automata.....	167
5.3.1 Conway's Game of Life .....	168
5.3.2 The Forest Fire Model .....	169
5.4 The Sandpile Model and Self-Organized Criticality .....	171
5.4.1 Absorbing Phase Transitions .....	173
5.5 Random Branching Theory .....	175
5.5.1 Branching Theory of Self-Organized Criticality .....	175
5.5.2 Galton-Watson Processes.....	180
5.6 Application to Long-Term Evolution .....	182
Exercises .....	188
Further Reading .....	189
References .....	190
<b>6 Darwinian Evolution, Hypercycles and Game Theory</b> .....	191
6.1 Introduction .....	191
6.2 Mutations and Fitness in a Static Environment .....	193
6.3 Deterministic Evolution .....	197
6.3.1 Evolution Equations .....	197
6.3.2 Beanbag Genetics: Evolutions Without Epistasis .....	200
6.3.3 Epistatic Interactions and the Error Catastrophe .....	202
6.4 Finite Populations and Stochastic Escape.....	206
6.4.1 Strong Selective Pressure and Adaptive Climbing .....	207
6.4.2 Adaptive Climbing Versus Stochastic Escape .....	210
6.5 Prebiotic Evolution.....	212
6.5.1 Quasispecies Theory.....	212
6.5.2 Hypercycles and Autocatalytic Networks .....	213
6.6 Macroecology and Species Competition .....	217
6.7 Coevolution and Game Theory .....	220
Exercises .....	225
Further Reading .....	226
References .....	227
<b>7 Synchronization Phenomena</b> .....	229
7.1 Frequency Locking.....	229
7.2 Synchronization of Coupled Oscillators .....	230
7.3 Synchronization with Time Delays.....	237
7.4 Synchronization via Aggregate Averaging.....	239
7.5 Synchronization via Causal Signaling .....	243
7.6 Synchronization and Object Recognition in Neural Networks .....	246
7.7 Synchronization Phenomena in Epidemics .....	250

Exercises .....	253
Further Reading .....	254
References .....	255
<b>8 Elements of Cognitive Systems Theory .....</b>	<b>257</b>
8.1 Introduction .....	257
8.2 Foundations of Cognitive Systems Theory .....	260
8.2.1 Basic Requirements for the Dynamics .....	260
8.2.2 Cognitive Information Processing Versus Diffusive Control .....	263
8.2.3 Basic Layout Principles .....	266
8.2.4 Learning and Memory Representations .....	267
8.3 Polyhomeostasis and Diffusive Emotional Control .....	271
8.3.1 Metalearning Through Polyhomeostasis .....	272
8.3.2 Emotional and Neutral Diffusive Control .....	274
8.4 Competitive Dynamics and Winning Coalitions .....	276
8.4.1 General Considerations .....	276
8.4.2 Associative Thought Processes .....	281
8.4.3 Autonomous Online Learning .....	285
8.5 Environmental Model Building .....	287
8.5.1 The Elman Simple Recurrent Network .....	287
8.5.2 Universal Prediction Tasks .....	292
Exercises .....	294
Further Reading .....	295
References .....	296
<b>9 Solutions .....</b>	<b>299</b>
9.1 Solutions to the Exercises of Chapter 1 .....	299
9.2 Solutions to the Exercises of Chapter 2 .....	306
9.3 Solutions to the Exercises of Chapter 3 .....	310
9.4 Solutions to the Exercises of Chapter 4 .....	316
9.5 Solutions to the Exercises of Chapter 5 .....	320
9.6 Solutions to the Exercises of Chapter 6 .....	325
9.7 Solutions to the Exercises of Chapter 7 .....	329
9.8 Solutions to the Exercises of Chapter 8 .....	332
<b>Index .....</b>	<b>335</b>