

Contents

| | |
|---|------|
| List of Tables | xiii |
| Notation and Conventions | xv |
| 1 Polynomial Ideals and Their Varieties | 1 |
| 1.1 Fundamental Concepts | 1 |
| 1.2 The Ideal Membership Problem and Gröbner Bases | 7 |
| 1.3 Basic Properties and Algorithms | 24 |
| 1.4 Decomposition of Varieties | 38 |
| 1.5 Notes and Complements | 50 |
| Exercises | 51 |
| 2 Stability and Normal Forms | 57 |
| 2.1 Lyapunov's Second Method | 57 |
| 2.2 Real Normal Forms | 62 |
| 2.3 Analytic and Formal Normal Forms | 68 |
| 2.4 Notes and Complements | 83 |
| Exercises | 84 |
| 3 The Center Problem | 89 |
| 3.1 The Poincaré First Return Map and the Lyapunov Numbers | 91 |
| 3.2 Complexification of Real Systems, Normal Forms, and the Center Problem | 96 |
| 3.3 The Center Variety | 108 |
| 3.4 Focus Quantities and Their Properties | 118 |
| 3.5 Hamiltonian and Reversible Systems | 128 |
| 3.6 Darboux Integrals and Integrating Factors | 136 |
| 3.7 Applications: Quadratic Systems and a Family of Cubic Systems ... | 147 |
| 3.8 The Center Problem for Liénard Systems | 158 |
| 3.9 Notes and Complements | 164 |
| Exercises | 167 |

| | | |
|----------|--|-----|
| 4 | The Isochronicity and Linearizability Problems | 175 |
| 4.1 | The Period Function | 175 |
| 4.2 | Isochronicity Through Normal Forms and Linearizability | 177 |
| 4.3 | The Linearizability Quantities | 191 |
| 4.4 | Darboux Linearization | 199 |
| 4.5 | Linearizable Quadratic Centers | 205 |
| 4.6 | Notes and Complements | 208 |
| | Exercises | 209 |
| 5 | Invariants of the Rotation Group | 213 |
| 5.1 | Properties of Invariants | 214 |
| 5.2 | The Symmetry Ideal and the Set of Time-Reversible Systems | 229 |
| 5.3 | Axes of Symmetry of a Plane System | 237 |
| 5.4 | Notes and Complements | 244 |
| | Exercises | 245 |
| 6 | Bifurcations of Limit Cycles and Critical Periods | 249 |
| 6.1 | Bautin's Method for Bifurcation Problems | 250 |
| 6.2 | The Cyclicity Problem | 257 |
| 6.3 | The Cyclicity of Quadratic Systems and a Family of Cubic Systems | 269 |
| 6.4 | Bifurcations of Critical Periods | 287 |
| 6.5 | Notes and Complements | 299 |
| | Exercises | 301 |
| | Appendix | 307 |
| | References | 313 |
| | Index of Notation | 323 |
| | Index | 327 |