

Contents

1	Exposing Methods: The 2014 European Parliament Elections	1
1.1	The 28 Member States of the 2014 Union	1
1.2	Austria, Belgium, Bulgaria; Electoral Keys	3
1.3	Cyprus, Czech Republic, Germany; Table Design	7
1.4	Denmark, Estonia, Greece; Alliances and Indeps	11
1.5	Spain, Finland, France, Croatia; Vote Patterns and Vote Categories	13
1.6	Hungary, Ireland, Italy; Quotas	19
1.7	Lithuania, Luxembourg, Latvia; Residual Fits	24
1.8	Malta, the Netherlands, Poland; Nested Stages	27
1.9	Portugal, Romania, Sweden; Method Overview	28
1.10	Slovenia, Slovakia, United Kingdom; Local Representation	33
1.11	Diversity Versus Uniformity	37
2	Imposing Constitutionality: The 2009 Bundestag Election	41
2.1	The German Federal Election Law	41
2.2	Countrywide Super-Apportionment 2009	44
2.3	Per-Party Sub-Apportionments 2009	46
2.4	Negative Voting Weights	48
2.5	Direct and Universal Suffrage	49
2.6	Free, Equal, and Secret Ballots	51
2.7	Equality of the Voters' Success Values	52
2.8	Equality of Representative Weights	54
2.9	Satisfaction of the Parties' Ideal Shares of Seats	55
2.10	Continuous Fits Versus Discrete Apportionments	56
3	From Reals to Integers: Rounding Functions and Rounding Rules	59
3.1	Rounding Functions	59
3.2	Floor Function	60
3.3	Ties and the Need for Rules of Rounding	60
3.4	Rule of Downward Rounding	61

3.5	Ceiling Function and Rule of Upward Rounding	62
3.6	Commercial Rounding Function	63
3.7	Rule of Standard Rounding	63
3.8	Signpost Sequences	65
3.9	Rounding Rules	66
3.10	Stationary Signposts	68
3.11	Power-Mean Signposts	68
3.12	Simple Rounding Does Not Suffice!	69
4	Divisor Methods of Apportionment: Divide and Round	71
4.1	House Size, Vote Weights, and Seat Numbers	71
4.2	Apportionment Rules	73
4.3	The Five Organizing Principles	75
4.4	Apportionment Methods	77
4.5	Divisor Methods	77
4.6	Max-Min Inequality	80
4.7	Jump-and-Step Procedure and the Select Divisor	82
4.8	Uniqueness, Multiplicities, and Ties	83
4.9	Tie Resolution Provisions	85
4.10	Primal Algorithms and Dual Algorithms	86
4.11	Adjusted Initialization for Stationary Divisor Methods	87
4.12	Universal Initialization	88
4.13	Bad Initialization	89
4.14	Highest Comparative Scores	91
4.15	Authorities	92
5	Quota Methods of Apportionment: Divide and Rank	95
5.1	Quota Methods	95
5.2	Hare-Quota Method with Residual Fit by Greatest Remainders	96
5.3	Greatest Remainders and the Select Split	97
5.4	Shift-Quota Methods	97
5.5	Max-Min Inequality	98
5.6	Shift-Quota Methods and Stationary Divisor Methods	100
5.7	Authorities	100
5.8	Quota Variants	101
5.9	Residual Fit Variants	102
5.10	Quota Method Variants	103
6	Targeting the House Size: Discrepancy Distribution	107
6.1	Seat Total and Discrepancy	107
6.2	Universal Divisor Initialization	108
6.3	Recommended Divisor Initialization	109
6.4	Distributional Assumptions	111
6.5	Seat-Total Distributions	113
6.6	Hagenbach-Bischoff Initialization	114

6.7	Discrepancy Probabilities: Formulas	116
6.8	Discrepancy Probabilities: Practice	118
6.9	Discrepancy Representation by Means of Rounding Residuals	121
6.10	Invariance Principle for Rounding Residuals	122
6.11	Discrepancy Distribution	124
7	Favoring Some at the Expense of Others: Seat Biases	127
7.1	Seat Excess of a Party	127
7.2	Rank-Order of Parties by Vote Shares	128
7.3	Vote Share Thresholds	128
7.4	Seat Bias Formula	129
7.5	Biasedness Versus Unbiasedness	130
7.6	House Size Recommendation	132
7.7	Cumulative Seat Biases by Subdivisions into Districts	133
7.8	Total Positive Bias: The Stronger Third, the Weaker Two-Thirds	134
7.9	Alliances of Lists	135
7.10	Proof of the Seat Bias Formula	139
7.11	Proof of the Seat Bias Formula for List Alliances	144
7.12	Seat Biases of Shift-Quota Methods	145
8	Preferring Stronger Parties to Weaker Parties: Majorization	149
8.1	Bipartitions by Vote Strengths	149
8.2	Majorization of Two Seat Vectors	150
8.3	A Sufficient Condition via Pairwise Comparisons	151
8.4	Majorization of Two Apportionment Methods	151
8.5	Majorization of Divisor Methods	152
8.6	Majorization-Increasing Parameterizations	153
8.7	Majorization Paths	154
8.8	Majorization of Shift-Quota Methods	156
9	Securing System Consistency: Coherence and Paradoxes	159
9.1	The Whole and Its Parts	159
9.2	The Sixth Organizing Principle: Coherence	160
9.3	Coherence and Completeness of Divisor Methods	161
9.4	Coherence Theorem	162
9.5	House Size Monotonicity	163
9.6	Vote Ratio Monotonicity	165
9.7	Retrieval of an Underlying Signpost Sequence	167
9.8	Proof of the Coherence Theorem in Sect. 9.4	168
9.9	Coherence and Stationary Divisor Methods	172
9.10	Coherence and the Divisor Method with Standard Rounding	175
9.11	Violation of Coherence: New States Paradox	177
9.12	Violation of House Size Monotonicity: Alabama Paradox	179

9.13	Violation of Vote Ratio Monotonicity: Population Paradox	181
9.14	Violation of Voter Monotonicity: No-Show Paradox.....	183
10	Appraising Electoral Equality: Goodness-of-Fit Criteria.....	185
10.1	Optimization of Goodness-of-Fit Criteria	185
10.2	Voter Orientation: DivStd	186
10.3	Curtailment of Overrepresentation: DivDwn	189
10.4	Alleviation of Underrepresentation: DivUpw	191
10.5	Parliamentary Orientation: DivGeo	193
10.6	Party Orientation: HaQgrR	195
10.7	Stabilization of Disparity Functions	198
10.8	Success-Value Stability: DivStd.....	199
10.9	Representative-Weight Stability: DivHar	200
10.10	Unworkable Disparity Functions.....	201
10.11	Ideal-Share Stability: DivStd	202
10.12	Ideal Share of Seats Versus Exact Quota of Seats	203
11	Tracing Peculiarities: Vote Thresholds and Majority Clauses.....	207
11.1	Vote Share Variation for a Given Seat Number.....	207
11.2	Vote Share Bounds: General Divisor Methods	208
11.3	Vote Share Bounds: Stationary Divisor Methods.....	209
11.4	Vote Share Bounds: Modified Divisor Methods	211
11.5	Vote Share Bounds: Shift-Quota Methods	212
11.6	Overview of Vote Thresholds	213
11.7	Preservation of a Straight Majority and Majority Clauses	215
11.8	House-Size Augmentation Clause	217
11.9	Majority-Minority Partition Clause	217
11.10	The 2002 German Conference Committee Dilemma	219
11.11	Residual-Seat Redirection Clause	220
11.12	Divisor Methods and Ideal Regions of Seats	221
12	Truncating Seat Ranges: Minimum-Maximum Restrictions	225
12.1	Minimum Representation for Electoral Districts	225
12.2	Quota Method Ambiguities	226
12.3	Minimum-Maximum Restricted Variants of Divisor Methods ...	227
12.4	Direct-Seat Restricted Variant of DivDwn.....	228
12.5	Proportionality Loss	232
12.6	Direct-Seat Restricted Variant of DivStd	232
12.7	Composition of the EP: Legal Requirements	234
12.8	Cambridge Compromise.....	236
12.9	Power Compromise	238
12.10	Jagiellonian Compromise.....	242
13	Proportionality and Personalization: BWG 2013.....	247
13.1	The 2013 Amendment of the Federal Election Law	247
13.2	Apportionment of Seats Among Parties	248
13.3	Assignment of Candidates to Seats	249

13.4	Initial Adjustment of the Bundestag Size	251
13.5	Alternative House Size Adjustment Strategies	252
14	Representing Districts and Parties: Double Proportionality	259
14.1	Double Proportionality: Practice	259
14.2	The 2016 Parliament Election in the Canton of Schaffhausen	260
14.3	Discordant Seat Assignments	264
14.4	Winner-Take-One Modification	265
14.5	Double Proportionality in Swiss Cantons: The Reality	265
14.6	Double Proportionality in the European Union: A Vision	266
14.7	Degressive Compositions and Separate District Evaluations	267
14.8	Compositional Proportionality for Unionwide Lists	268
15	Double-Proportional Divisor Methods: Technicalities	275
15.1	Row and Column Marginals, Weight and Seat Matrices	275
15.2	Double-Proportional Divisor Methods	276
15.3	Cycles of Seat Transfers	278
15.4	Uniqueness of Double-Proportional Seat Matrices	279
15.5	Characterization of Double-Proportional Seat Matrices	281
15.6	Existence of Double-Proportional Seat Matrices	285
15.7	A Dual View	287
15.8	Alternating Scaling Algorithm	288
15.9	Tie-and-Transfer Algorithm	292
16	Biographical Digest	297
16.1	Thomas Jefferson 1743–1826	297
16.2	Alexander Hamilton 1755–1804	298
16.3	Daniel Webster 1782–1852	299
16.4	Thomas Hare 1806–1891	300
16.5	Henry Richmond Droop 1832–1884	301
16.6	Eduard Hagenbach-Bischoff 1833–1910	302
16.7	Victor D’Hondt 1841–1901	305
16.8	Joseph Adna Hill 1860–1938	306
16.9	Walter Francis Willcox 1861–1964	308
16.10	Ladislaus von Bortkiewicz 1868–1931	309
16.11	Edward Vermilye Huntington 1874–1952	310
16.12	Siegfried Geyerhahn 1879–1960	312
16.13	Jean-André Sainte-Laguë 1882–1950	313
16.14	George Pólya 1887–1985	315
16.15	Horst Friedrich Niemeyer 1931–2007	317
	Notes and Comments	319
	References	331
	Index	339