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H. Ruder (Eds.)

Relativity and Scientific Computing

Computer Algebra,
Numerics, Visualization

With 55 Figures, 25 Color Plates,
and 11 Tables



Springer

Contents

Color Plates	1
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Part I: Numerics

1. Numerical Relativity and Black-Hole Collisions

Edward Seidel	25
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1. Astrophysical Motivation and Mathematical Formulation for Numerical Relativity	25
1.1 Overview and Motivation	25
1.2 Mathematical Formulation of the Equations	26
2. Numerical Techniques and Supercomputing	32
2.1 Finite Difference Techniques	32
2.2 Treating Elliptic Equations	33
2.3 Treating Evolution Equations	35
2.4 Testbeds and Convergence of Numerical Solutions	37
2.5 Coding and Parallel-Computing Issues	38
2.6 General Code Strategies	38
3. Black-Hole Initial-Data Sets, Tools for Analysis, and Techniques for Evolution	40
3.1 Basic Theory and Initial-Data Sets	41
3.2 Tools for Numerical Black-Hole Spacetimes	46
3.3 Evolution	55
4. Present Research Status in Black-Hole Studies	59
4.1 Spherical BH – 1D	59
4.2 Distorted BH – 2D	59
4.3 Rotating BH – 2D	60
4.4 Colliding BH – 2D	61
4.5 Black-Hole Horizon Studies	62
4.6 3D Black-Hole Studies	63
References	64

2. Four Lectures on Numerical Relativity

Carles Bona	69
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1. The Causal Structure of Einstein's Field Equations	69
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VIII Contents

1.1	The Space-Plus-Time Decomposition	69
1.2	Invariant Algebraic Slicing	70
1.3	The Evolution System	71
1.4	Causal Structure of the Evolution System	73
2.	First-Order Flux-Conservative Systems	74
2.1	Linear Systems	75
2.2	Nonlinear Systems	76
2.3	Einstein's Evolution Equations	76
3.	Standard Numerical Methods	78
3.1	Flux-Conservative Equations	79
3.2	Boundary Conditions	81
3.3	Nonsmooth Data	82
4.	Total Variation Diminishing Methods	82
4.1	Flux-Conservative Methods	84
4.2	The 1D Black-Hole Test	85
	References	87
 3. Alternatives to Finite Difference Methods in Numerical Relativity		
Pablo Laguna	88	
1.	Introduction	88
2.	The 3+1 Formalism	90
3.	The Initial-Data Problem	92
3.1	Multiquadratics	93
3.2	Finite Elements	96
4.	Matter Evolution in Curved Spacetimes	98
4.1	Particle-Mesh Methods	98
4.2	Smoothed Particle Hydrodynamics in Curved Space	101
5.	Conclusions	107
	References	108
 4. Temporal and Spatial Foliations of Spacetimes		
Heinz Herold	111	
1.	Introduction	111
2.	Time Slicings	112
2.1	Maximal Slicing	113
2.2	Harmonic Slicing	113
2.3	Results for the Oppenheimer-Snyder Spacetime	114
3.	Spatial Foliations	117
3.1	Constant Mean Curvature Foliations	118
3.2	Examples of CMC Foliations	119
	References	120

5. Rotating and Oscillating Neutron Stars	
Heinz Herold	122
1. Introduction: Nonrotating Neutron Stars	122
2. Rotating Neutron Stars	123
2.1 Basic Formulation.....	124
2.2 Numerical Solution Method	126
2.3 Results	128
3. Oscillations of Neutron Stars	132
3.1 Basic Formalism	132
3.2 Results for Quasinormal Mode Frequencies	134
References	136
6. Rotating Boson Stars	
Franz E. Schunck and Eckehard W. Mielke	138
1. Introduction	138
2. Field Equations	140
3. Spherically Symmetric Boson Star	140
4. Field Equations of a Rotating Boson Star	142
5. Particle Number, Mass, and Angular Momentum.....	144
6. Differential Rotation	145
7. Numerical Solution	146
8. Remarks	147
References	150
7. Numerical Investigation of Cosmological Singularities	
Beverly K. Berger	152
1. Introduction	152
2. Symplectic Methods	155
3. Mixmaster Model	157
4. Gowdy Model on $T^3 \times R$	159
5. U(1) Models	162
6. Conclusions	165
Appendix	165
References	168
Part II: Computer Algebra	
8. Overview of Computer Algebra in Relativity	
David Hartley	173
1. Introduction	173
2. General-Purpose Systems	174
2.1 Computer Algebra	174
2.2 Modern Systems	175

2.3 Evaluation	176
2.4 Simplification	177
2.5 Programming	180
2.6 Data Types	181
3. General Relativity Systems	182
3.1 Requirements: Riemannian Geometry	182
3.2 Requirements: GR Library	184
3.3 Requirements: Generalizations and Applications	184
3.4 Efficiency	185
3.5 Dummy Indices	187
4. Applications	188
5. Summary	189
References	189
 9. Two-Loop Quantum Gravity with the Computer Algebra Program FORM	
Anton E.M. van de Ven	192
1. Introduction	192
2. Covariant Quantization of Gravity	193
2.1 Noncovariant Method	195
2.2 Covariant Method	196
3. FORM	199
3.1 Annotated Yang-Mills Program	200
3.2 Quantum Gravity	206
References	209
 10. The Mathematic Packages CARTAN and MathTensor for Tensor Analysis	
Harald H. Soleng	210
1. Mathematica	210
1.1 The Front End	211
1.2 The Kernel	211
2. Tensor Calculations	212
3. CARTAN	213
3.1 General Features	213
3.2 A Charged Black Hole	214
3.3 A Spin-Polarized Cosmic String	219
4. MathTensor	223
4.1 Tensor Indices	223
4.2 Metric Variation of R^2	223
4.3 Differential Forms with MathTensor	226
5. Conclusions	228
Appendix	229
References	229

11. A Quadratic Curvature Lagrangian of Pawłowski and Rączka: A Finger Exercise with MathTensor	
Efstratios Tsantilis, Roland A. Puntigam, and Friedrich W. Hehl.....	231
1. Introduction.....	231
2. Riemann Tensor and its Irreducible Pieces	232
3. The Topological Euler Density	233
4. Bach Tensor	235
5. The Bach Tensor Streamlined	236
6. Gravitational Field Equation of the P&R Model	237
7. Discussion	239
References	239
12. The Program CRACK for Solving PDEs in General Relativity	
Thomas Wolf.....	241
1. Introduction.....	241
2. Contents of CRACK.....	243
2.1 General Remarks	243
2.2 Decoupling	243
2.3 Integrating Exact PDEs	244
2.4 Separation of PDEs	246
2.5 Solving Standard ODEs.....	246
3. The Calculation of Spacetime Symmetries.....	246
3.1 General Remarks	246
3.2 Generation of the Conditions	247
3.3 Post Processing of Results	249
3.4 An Example.....	249
4. Symmetries of a Field Theory	251
5. Applying Symmetries of Differential Equations.....	252
6. Future Work	253
6.1 General Remarks	253
6.2 Extending Capabilities in Dealing with DEs	254
6.3 Advanced Symmetry Investigations.....	255
7. Availability	257
References	257
13. Algebraic Programming in the Hamiltonian Version of General Relativity	
Dumitru N. Vulcanov.....	259
1. Introduction.....	259
2. The Local Form of the Canonical Formalism of Gravity	260
3. The Computer Procedures	263
4. About the Concrete Results	266

4.1 Spacetime Model with a Three Subspace in Form of a 3-Torus	266
References	268

14. Causal Structure and Integrability in Moving Frames with Reduce

David Hartley	270
1. Introduction	270
2. Exterior Differential Systems	271
3. EDS Package	274
4. Ricci-Flat Immersions	276
5. Twisting Type-N Solutions	279
References	283

Part III: Visualization

15. Four Lectures on Computer Graphics and Data Visualization

G. David Kerlick	287
1. Introduction	287
2. Computers, Humans, and Interaction	288
2.1 Computer Graphics Defined	288
2.2 Computer Displays	288
2.3 Graphics Hardware	289
2.4 Human Visual System	290
2.5 History of Computer Graphics	291
3. Visual Simulation	292
3.1 Visibility Calculation	292
3.2 Simulated Optics	293
3.3 Ray-Tracing	294
3.4 Rasterization	295
3.5 Graphics Pipeline	295
4. Data Visualization	295
4.1 Seeing the Invisible	295
4.2 The Analysis and Visualization Cycles	296
4.3 History of Visualization	296
4.4 Data Sources and Structure	296
4.5 Data Interpolation	297
5. Visualization Techniques	298
5.1 Scalar Fields	298
5.2 Vector Fields	299
5.3 Tensor Fields	300
5.4 Differential Geometry Applied to Height Maps	301
5.5 Second Derivatives and Curvature	302
5.6 Visualizing Higher Dimensions	303

5.7 How to Lie and Confuse with Visualization	303
6. Interactive Visualization Environments	303
6.1 Data Structures.....	304
6.2 Modules	304
6.3 Visual Programming Language and Execution Model	304
6.4 Programmers' Tools	305
7. Virtual Environments in Visualization	305
7.1 Visualization over the World Wide Web.....	306
8. Applications and Demonstrations	306
8.1 Case Study: Structural Dynamics Using DYNASTD	307
8.2 Case Study: Helicopter Rotor Wake Visualization	307
8.3 An Example of Remote Collaboration	308
9. Conclusions	309
References	310

16. Visualization in Curved Spacetimes.**I. Visualization of Objects via Four-Dimensional Ray-Tracing**

Hans-Peter Nollert, Ute Kraus, and Hanns Ruder	314
1. Introduction	314
2. Ray-Tracing Special Relativity.....	315
2.1 Geometrical Appearance	315
2.2 Lighting	319
3. Ray-Tracing General Relativity	321
3.1 Requirements	322
3.2 Geodesic Equation	322
3.3 Camera	323
3.4 Realizing Relativistic Ray-Tracing with Conventional Ray-Tracing Programs	323
3.5 Astrophysical Examples.....	325
3.6 Astrophysical Application: Light Curves of X-Ray Pulsars.....	327
4. Discussion	327
References	329

17. Visualization in Curved Spacetimes.**II. Visualization of Surfaces via Embedding**

Hans-Peter Nollert and Heinz Herold	330
1. Introduction	330
2. Finding the Parametric Representation of the Embedding in R^3 ..	331
2.1 Useful Results from Differential Geometry of Surfaces	331
2.2 The System of Equations	333
2.3 An Alternative: The Darboux Equation	336
3. Examples	338
3.1 Kerr Horizon	338
3.2 Colliding Black Holes	341

4. Direct Construction of Wire Frames	343
4.1 Triangular Wire Frames	344
4.2 Surfaces with Spherical Topology	344
4.3 Procedure	348
4.4 Examples	349
4.5 Future Developments	350
References	351

Part IV: Exotica

18. Exotic Smoothness on Spacetime

Carl H. Brans	355
1. Introduction	355
2. Differential Topology	356
3. Early Exotica of Various Kinds	360
4. Gauge Theory and Moduli Spaces	363
5. The Road to \mathbf{R}_Θ^4	366
6. Some General Properties of \mathbf{R}_Θ^4	367
7. Some Geometry and Physics on \mathbf{R}_Θ^4 s	369
8. Conclusions	375
References	376
List of Figures	379
List of Tables	382
Index	383