Gregory C. Reinsel

Elements of Multivariate Time Series Analysis

÷.

Second Edition

With 14 Figures

۰._



Contents

Preface to the Second Edition	vii
Preface to the First Edition	ix
1. Vector Time Series and Model Representations	1
1.1 Stationary Multivariate Time Series and Their Properties	2
1.1.1 Covariance and Correlation Matrices for a Stationary Vector Process	2
1.1.2 Some Spectral Characteristics for a Stationary Vector Process	4
1.1.3 Some Relations for Linear Filtering of a Stationary Vector Process	5
1.2 Linear Model Representations for a Stationary Vector Process	7
1.2.1 Infinite Moving Average (Wold) Representation of a Stationary Vector Process	7
1.2.2 Vector Autoregressive Moving Average (ARMA) Model Representations	7
A1 Appendix: Review of Multivariate Normal Distribution and Related Topics	12
A1.1 Review of Some Basic Matrix Theory Results	12
A1.2 Vec Operator and Kronecker Products of Matrices	13
A1.3 Expected Values and Covariance Matrices of Random Vectors	14
A1.4 The Multivariate Normal Distribution	14
A1.5 Some Basic Results on Stochastic Convergence	19
2. Vector ARMA Time Series Models and Forecasting	22
2.1 Vector Moving Average Models	22
2.1.1 Invertibility of the Vector Moving Average Model	22

2.1.2	Covariance Matrices of the Vector Moving Average Model	2
2.1.3	Features of the Vector MA(1) Model	2
	Model Structure for Subset of Components in the Vector	_
	MA Model	2
2.2 Ve	ctor Autoregressive Models	2
	Stationarity of the Vector Autoregressive Model	2
	Yule–Walker Relations for Covariance Matrices of a Vector AR Process	2
2.2.3	Covariance Features of the Vector AR(1) Model	2
	Univariate Model Structure Implied by Vector AR Model	3
	ctor Mixed Autoregressive Moving Average Models	
	Stationarity and Invertibility of the Vector ARMA Model	
	Relations for the Covariance Matrices of the Vector ARMA Model	-
2.3.3	Some Features of the Vector ARMA(1,1) Model	-
	Consideration of Parameter Identifiability for Vector ARMA Models	
2.3.5	Further Aspects of Nonuniqueness of Vector ARMA Model Representations	
2.4 No	onstationary Vector ARMA Models	
	Vector ARIMA Models for Nonstationary Processes	
	Cointegration in Nonstationary Vector Processes	
	The Vector IMA(1,1) Process or Exponential Smoothing Model	
2.5 Pre	ediction for Vector ARMA Models	
-	Minimum Mean Squared Error Prediction	
	Forecasting for Vector ARMA Processes and Covariance Matrices of Forecast Errors	
253	Computation of Forecasts for Vector ARMA Processes	
	Some Examples of Forecast Functions for Vector ARMA	
2.5.4	Models	
2.6 St	ate-Space Form of the Vector ARMA Model	
	ppendix: Methods for Obtaining Autoregressive and	
	Ioving Average Parameters from Covariance Matrices	
	Iterative Algorithm for Factorization of Moving Average	
112.1	Spectral Density Matrix in Terms of Covariance Matrices	
A2.2	Autoregressive and Moving Average Parameter Matrices	
	in Terms of Covariance Matrices for the Vector	
	ARMA Model	
A2.3	Evaluation of Covariance Matrices in Terms of the AR and MA Parameters for the Vector ARMA Model	

Contents

3.	Canoni	cal Structure of Vector ARMA Models	
		onsideration of Kronecker Structure for Vector ARMA	
		Kronecker Indices and McMillan Degree of Vector ARMA Process	
	3.1.2	Echelon Form Structure of Vector ARMA Model Implied by Kronecker Indices	
	3.1.3	Reduced-Rank Form of Vector ARMA Model Implied by Kronecker Indices	
	3.2 Ca	nonical Correlation Structure for ARMA Time Series	
	3.2.1	Review of Canonical Correlations in Multivariate Analysis	
	3.2.2	Canonical Correlations for Vector ARMA Processes	
	3.2.3	Relation to Scalar Component Model Structure	
		tial Autoregressive and Partial Correlation Matrices	
		Vector Autoregressive Model Approximations and Partial Autoregression Matrices	
	3.3.2	Recursive Fitting of Vector AR Model Approximations	
	3.3.3	Partial Cross-Correlation Matrices for a Stationary Vector Process	
	3.3.4	Partial Canonical Correlations for a Stationary Vector Process	
4.	Initial Model	Model Building and Least Squares Estimation for Vector AR s	
	4.1 Sa P	mple Cross-Covariance and Correlation Matrices and Their roperties	
	4.1.1	Sample Estimates of Mean Vector and of Covariance and Correlation Matrices	
	4.1.2	Asymptotic Properties of Sample Correlations	
		Imple Partial AR and Partial Correlation Matrices and Their roperties	
	4.2.1	Test for Order of AR Model Based on Sample Partial Autoregression Matrices	
	4.2.2	Equivalent Test Statistics Based on Sample Partial Correlation Matrices	
	4.3 Co	nditional Least Squares Estimation of Vector AR Models	
		Least Squares Estimation for the Vector AR(1) Model	
		Least Squares Estimation for the Vector AR Model of General Order	
	4.3.3	Likelihood Ratio Testing for the Order of the AR Model	

	Derivation of the Wald Statistic for Testing the Order of the AR Model	9
4.4 Re	lation of LSE to Yule–Walker Estimate for Vector AR odels	. 9
4.5 Ad	ditional Techniques for Specification of Vector ARMA odels	10
4.5.1	Use of Order Selection Criteria for Model Specification	10
4.5.2	Sample Canonical Correlation Analysis Methods	10
4.5.3	Order Determination Using Linear LSE Methods for the Vector ARMA Model	10
	pendix: Review of the General Multivariate Linear egression Model	11
A4.1	Properties of the Maximum Likelihood Estimator of the Regression Matrix	11
A4.2	Likelihood Ratio Test of Linear Hypothesis About Regression Coefficients	11
A4.3	Asymptotically Equivalent Forms of the Test of Linear Hypothesis	11
A4.4	Multivariate Linear Model with Reduced-Rank Structure	11
A4.5	Generalization to Seemingly Unrelated Regressions Model	12
	um Likelihood Estimation and Model Checking for Vector Models	12
5.1 Co	onditional Maximum Likelihood Estimation for Vector	
A	RMA Models	12
5.1.1	Conditional Likelihood Function for the Vector ARMA Model	12
5.1.2	Likelihood Equations for Conditional ML Estimation	12
	Iterative Computation of the Conditional MLE by GLS Estimation	12
5.1.4	Asymptotic Distribution for the MLE in the Vector ARMA Model	12
5.2 M	L Estimation and LR Testing of ARMA Models Under	
	inear Restrictions	13
5.2.1	ML Estimation of Vector ARMA Models with Linear	
600	Constraints on the Parameters	1: 1:
	LR Testing of the Hypothesis of the Linear Constraints ML Estimation of Vector ARMA Models in the Echelon	
	Canonical Form	1
5.3 Ex	act Likelihood Function for Vector ARMA Models	1

5.3.1	Expressions for the Exact Likelihood Function and Exact Backcasts	13
5.3.2	Special Cases of the Exact Likelihood Results	13
	Finite Sample Forecast Results Based on the Exact Likelihood Approach	14
	novations Form of the Exact Likelihood Function for RMA Models	14
	Use of Innovations Algorithm Approach for the Exact Likelihood	14
5.4.2	Prediction of Vector ARMA Processes Using the Innovations Approach	14
5.5 Ov	verall Checking for Model Adequacy	14
5.5.1	Residual Correlation Matrices and Overall Goodness-of- Fit Test	14
5.5.2	Asymptotic Distribution of Residual Covariances and Goodness-of-Fit Statistic	1.
5.5.3	Use of the Score Test Statistic for Model Diagnostic Checking	1:
	ffects of Parameter Estimation Errors on Prediction roperties	1:
5.6.1	Effects of Parameter Estimation Errors on Forecasting in the Vector $AR(p)$ Model	1:
5.6.2	Prediction Through Approximation by Autoregressive Model Fitting	1:
	lotivation for AIC as Criterion for Model Selection, and Corrected Versions of AIC	10
	umerical Examples	1
	ed-Rank and Nonstationary Cointegrated Models	1′
	ested Reduced-Rank AR Models and Partial Canonical Correlation Analysis	1
6.1.1	Specification of Ranks Through Partial Canonical Correlation Analysis	1
6.1.2	Canonical Form for the Reduced-Rank Model	1
6.1.3	Maximum Likelihood Estimation of Parameters in the Model	1
6.1.4	Relation of Reduced-Rank AR Model with Scalar Component Models and Kronecker Indices	1
	eview of Estimation and Testing for Nonstationarity (Unit Roots) in Univariate ARIMA Models	1
	Limiting Distribution Results in the AR(1) Model with a Unit Root	1

6.2.2	Unit-Root Distribution Results for General Order AR Models	1
	onstationary (Unit-Root) Multivariate AR Models, stimation, and Testing	1
	Unit-Root Nonstationary Vector AR Model, the Error- Correction Form, and Cointegration	1
6.3.2	Asymptotic Properties of the Least Squares Estimator	1
6.3.3	Reduced-Rank Estimation of the Error-Correction Form of the Model	1
6.3.4	Likelihood Ratio Test for the Number of Unit Roots	1
6.3.5	Reduced-Rank Estimation Through Partial Canonical Correlation Analysis	2
6.3.6	Extension to Account for a Constant Term in the Estimation	2
6.3.7	Forecast Properties for the Cointegrated Model	2
6.3.8	Explicit Unit-Root Structure of the Nonstationary AR Model and Implications	2
6.3.9	Further Numerical Examples	2
	Canonical Analysis for Vector Autoregressive Time eries	2
6.4.1	Canonical Analysis Based on Measure of Predictability	2
6.4.2	Application to the Analysis of Nonstationary Series for Cointegration	2
6.5 M	ultiplicative Seasonal Vector ARMA Models	2
	Some Special Seasonal ARMA Models for Vector Time Series	2
7. State-	Space Models, Kalman Filtering, and Related Topics	2
7.1 St	ate-Variable Models and Kalman Filtering	2
7.1.1	The Kalman Filtering Relations	2
7.1.2	Smoothing Relations in the State-Variable Model	2
7.1.3	Innovations Form of State-Space Model and Steady State for Time-Invariant Models	
7.1.4	Controllability, Observability, and Minimality for Time-Invariant Models	
7.2 St	ate-Variable Representations of the Vector ARMA Model	2
7.2.1	A State-Space Form Based on the Prediction Space of Future Values	2
7.2.2	Exact Likelihood Function Through the State-Variable Approach	2
7.2.3	Alternate State-Space Forms for the Vector ARMA Model	2

7.2.4 Minimal Dimension State-Variable Representation and Kronecker Indices	247
7.2.5 (Minimal Dimension) Echelon Canonical State-Space	247
Representation	247
7.3 Exact Likelihood Estimation for Vector ARMA Processes with Missing Values	255
7.3.1 State-Space Model and Kalman Filtering with Missing Values	255
7.3.2 Estimation of Missing Values in ARMA Models	257
7.3.3 Initialization for Kalman Filtering, Smoothing, and Likelihood Evaluation in Nonstationary Models	260
7.4 Classical Approach to Smoothing and Filtering of Time Series	265
7.4.1 Smoothing for Univariate Time Series	266
7.4.2 Smoothing Relations for the Signal Plus Noise or	
Structural Components Model	269
7.4.3 A Simple Vector Structural Component Model for Trend	272
8. Linear Models with Exogenous Variables	274
8.1 Representations of Linear Models with Exogenous Variables	274
8.2 Forecasting in ARMAX Models	
8.2.1 Forecasts When Future Exogenous Variables Must Be Forecasted	
8.2.2 MSE Matrix of Optimal Forecasts	
8.2.3 Forecasting When Future Exogenous Variables Are Specified	
8.3 Optimal Feedback Control in ARMAX Models	
8.4 Model Specification, ML Estimation, and Model Checking	
for ARMAX Models	285
8.4.1 Some Comments on Specification and Checking of ARMAX Models	285
8.4.2 ML Estimation for ARMAX Models	286
8.4.3 Asymptotic Distribution Theory of Estimators in	
ARMAX Models	289
8.5 Numerical Example	292
Appendix: Time Series Data Sets	299
Exercises and Problems	315
References	332
Subject Index	345
Author Index	354