Fluid Physics in Geology

An Introduction to Fluid Motions on Earth's Surface and Within Its Crust

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