Contents

Chapter 1. Introduction	1
1.1. Munk boundary layers	1
1.1.1. State of the art	2
1.1.2. Boundary layer degeneracies	3
1.1.3. Stability of the stationary Munk equation	3
1.2. Geometrical preliminaries	5
1.2.1. Regularity and flatness assumptions	5
1.2.2. Singularity lines	7
1.2.3. Domains with islands	8
1.2.4. Periodic domains and domains with corners	10
1.3. Main approximation results	11
1.3.1. General case	11
1.3.2. Periodic and rectangle cases	14
1.3.3. Outline of the paper	16
Chapter 2. Multiscale analysis	17
2.1. Local coordinates and the boundary layer equation	17
2.2. East and West boundary layers	19
2.2.1. The scaled equation	19
2.2.2. Domain of validity	20
2.3. North and South boundary layers	21
2.3.1. The scaled equation	21
2.3.2. Study of the boundary layer equation (2.11):	22
2.3.3. Boundary conditions for $s \in (s_i, s_{i+1})$	24
2.3.4. Connection with East and West boundary layers	25
2.4. Discontinuity zones	26
2.4.1. Lifting the discontinuity	28
2.4.2. The interior singular layer	31
2.5. The case of islands	32
2.6. North and South periodic boundary layers	35
Chapter 3. Construction of the approximate solution	37
3.1. The interior term	38
3.2. Lifting the East boundary conditions	-1-1
3.2.1. Traces of the East boundary layers	45
3.2.2. Definition of the East corrector	-46
3.3. North and South boundary layers	47
3.3.1. Definition of the initial boundary value problem	47
3.3.2. Estimates for $\psi_{N,S}$	50
3.3.3. Extinction and truncation	58

3.4. The interface layer	59
3.4.1. The lifting term ψ^{lift}	59
3.4.2. The interior singular layer ψ^{Σ}	60
3.4.3. Connection with the West boundary	64
3.5. Lifting the West boundary conditions	67
3.6. Approximate solution in the periodic and rectangle case	69
3.6.1. In the periodic case	69
3.6.2. In the rectangle case	69
Chapter 4. Proof of convergence	71
4.1. Remainders stemming from the interior term $\psi^{int} = \psi^0_t + \psi^{\text{lift}}$	71
4.1.1. Error terms due to the truncation $\chi_{\mathfrak{E}}$.	71
4.1.2. Error terms due to the lifting term ψ^{lift}	75
4.2. Remainders coming from the boundary terms	76
4.2.1. Laplacian in curvilinear coordinates	76
4.2.2. Error terms associated with North and South layers	79
4.2.3. Error terms associated with East and West boundary layers	82
4.2.4. Error terms associated with discontinuity layers	88
4.3. Remainders in the periodic and rectangular cases	88
Chapter 5. Discussion: Physical relevance of the model	91
Acknowledgments	93
Appendix	95
Appendix A: The case of islands: derivation of the compatibility condition	0.5
(1.15) and proof of Lemma 1.2.1	95
Appendix B: Equivalents for the coordinates of boundary points near	
horizontal parts	96
Appendix C: Estimates on the coefficients a and b .	91
Appendix D: Proof of Lemma 3.4.3	100
Notations	101
Sizes of parameters and terms	103
Bibliography	105