

Numerical
Techniques in

Electromagnetics

Second Edition

Matthew N. O. Sadiku, Ph.D.



CRC Press

Boca Raton London New York Washington, D.C.

Contents

| | | |
|----------|--|-----------|
| 1 | Fundamental Concepts | 1 |
| 1.1 | Introduction | 1 |
| 1.2 | Review of Electromagnetic Theory | 2 |
| 1.2.1 | Electrostatic Fields | 3 |
| 1.2.2 | Magnetostatic Fields | 4 |
| 1.2.3 | Time-varying Fields | 5 |
| 1.2.4 | Boundary Conditions | 7 |
| 1.2.5 | Wave Equations | 7 |
| 1.2.6 | Time-varying Potentials | 9 |
| 1.2.7 | Time-harmonic Fields | 10 |
| 1.3 | Classification of EM Problems | 14 |
| 1.3.1 | Classification of Solution Regions | 14 |
| 1.3.2 | Classification of Differential Equations | 15 |
| 1.3.3 | Classification of Boundary Conditions | 18 |
| 1.4 | Some Important Theorems | 20 |
| 1.4.1 | Superposition Principle | 20 |
| 1.4.2 | Uniqueness Theorem | 21 |
| | References | 23 |
| | Problems | 23 |
| 2 | Analytical Methods | 27 |
| 2.1 | Introduction | 27 |
| 2.2 | Separation of Variables | 28 |
| 2.3 | Separation of Variables in Rectangular Coordinates | 30 |
| 2.3.1 | Laplace's Equations | 30 |
| 2.3.2 | Wave Equation | 34 |

| | | |
|----------|--|------------|
| 2.4 | Separation of Variables in Cylindrical Coordinates | 39 |
| 2.4.1 | Laplace's Equation | 40 |
| 2.4.2 | Wave Equation | 42 |
| 2.5 | Separation of Variables in Spherical Coordinates | 53 |
| 2.5.1 | Laplace's Equation | 54 |
| 2.5.2 | Wave Equation | 59 |
| 2.6 | Some Useful Orthogonal Functions | 68 |
| 2.7 | Series Expansion | 78 |
| 2.7.1 | Poisson's Equation in a Cube | 78 |
| 2.7.2 | Poisson's Equation in a Cylinder | 80 |
| 2.7.3 | Strip Transmission Line | 83 |
| 2.8 | Practical Applications | 88 |
| 2.8.1 | Scattering by Dielectric Sphere | 88 |
| 2.8.2 | Scattering Cross Sections | 92 |
| 2.9 | Attenuation Due to Raindrops | 95 |
| 2.10 | Concluding Remarks | 105 |
| | References | 106 |
| | Problems | 107 |
| 3 | Finite Difference Methods | 121 |
| 3.1 | Introduction | 121 |
| 3.2 | Finite Difference Schemes | 122 |
| 3.3 | Finite Differencing of Parabolic PDEs | 125 |
| 3.4 | Finite Differencing of Hyperbolic PDEs | 131 |
| 3.5 | Finite Differencing of Elliptic PDEs | 134 |
| 3.5.1 | Band Matrix Method | 137 |
| 3.5.2 | Iterative Methods | 137 |
| 3.6 | Accuracy and Stability of FD Solutions | 143 |
| 3.7 | Practical Applications I — Guided Structures | 147 |
| 3.7.1 | Transmission Lines | 148 |
| 3.7.2 | Waveguides | 154 |
| 3.8 | Practical Applications II — Wave Scattering (FDTD) | 159 |
| 3.8.1 | Yee's Finite Difference Algorithm | 160 |
| 3.8.2 | Accuracy and Stability | 163 |
| 3.8.3 | Lattice Truncation Conditions | 164 |
| 3.8.4 | Initial Fields | 167 |
| 3.8.5 | Programming Aspects | 168 |
| 3.9 | Absorbing Boundary Conditions for FDTD | 177 |
| 3.10 | Finite Differencing for Nonrectangular Systems | 186 |
| 3.10.1 | Cylindrical Coordinates | 186 |
| 3.10.2 | Spherical Coordinates | 190 |
| 3.11 | Numerical Integration | 193 |
| 3.11.1 | Euler's Rule | 196 |
| 3.11.2 | Trapezoidal Rule | 197 |
| 3.11.3 | Simpson's Rule | 197 |

| | | |
|----------|--|------------|
| 3.11.4 | Newton-Cotes Rules | 198 |
| 3.11.5 | Gaussian Rules | 200 |
| 3.11.6 | Multiple Integration | 203 |
| 3.12 | Concluding Remarks | 208 |
| | References | 210 |
| | Problems | 219 |
| 4 | Variational Methods | 235 |
| 4.1 | Introduction | 235 |
| 4.2 | Operators in Linear Spaces | 236 |
| 4.3 | Calculus of Variations | 238 |
| 4.4 | Construction of Functionals from PDEs | 242 |
| 4.5 | Rayleigh-Ritz Method | 245 |
| 4.6 | Weighted Residual Method | 252 |
| 4.6.1 | Collocation Method | 253 |
| 4.6.2 | Subdomain Method | 254 |
| 4.6.3 | Galerkin Method | 254 |
| 4.6.4 | Least Squares Method | 255 |
| 4.7 | Eigenvalue Problems | 261 |
| 4.8 | Practical Applications | 268 |
| 4.9 | Concluding Remarks | 274 |
| | References | 275 |
| | Problems | 279 |
| 5 | Moment Methods | 285 |
| 5.1 | Introduction | 285 |
| 5.2 | Integral Equations | 286 |
| 5.2.1 | Classification of Integral Equations | 286 |
| 5.2.2 | Connection Between Differential and Integral Equations | 287 |
| 5.3 | Green's Functions | 290 |
| 5.3.1 | For Free Space | 292 |
| 5.3.2 | For Domain with Conducting Boundaries | 295 |
| 5.4 | Applications I — Quasi-Static Problems | 308 |
| 5.5 | Applications II — Scattering Problems | 313 |
| 5.5.1 | Scattering by Conducting Cylinder | 314 |
| 5.5.2 | Scattering by an Arbitrary Array of Parallel Wires | 317 |
| 5.6 | Applications III — Radiation Problems | 325 |
| 5.6.1 | Hallen's Integral Equation | 326 |
| 5.6.2 | Pocklington's Integral Equation | 327 |
| 5.6.3 | Expansion and Weighting Functions | 327 |
| 5.7 | Applications IV — EM Absorption in the Human Body | 338 |
| 5.7.1 | Derivation of Integral Equations | 339 |
| 5.7.2 | Transformation to Matrix Equation (Discretization) | 342 |
| 5.7.3 | Evaluation of Matrix Elements | 343 |
| 5.7.4 | Solution of the Matrix Equation | 345 |

| | | |
|----------|---|------------|
| 5.8 | Concluding Remarks | 347 |
| | References | 357 |
| | Problems | 363 |
| 6 | Finite Element Method | 377 |
| 6.1 | Introduction | 377 |
| 6.2 | Solution of Laplace's Equation | 378 |
| 6.2.1 | Finite Element Discretization | 378 |
| 6.2.2 | Element Governing Equations | 380 |
| 6.2.3 | Assembling of All Elements | 383 |
| 6.2.4 | Solving the Resulting Equations | 386 |
| 6.3 | Solution of Poisson's Equation | 397 |
| 6.3.1 | Deriving Element-governing Equations | 397 |
| 6.3.2 | Solving the Resulting Equations | 399 |
| 6.4 | Solution of the Wave Equation | 400 |
| 6.5 | Automatic Mesh Generation I — Rectangular Domains | 407 |
| 6.6 | Automatic Mesh Generation II — Arbitrary Domains | 410 |
| 6.6.1 | Definition of Blocks | 411 |
| 6.6.2 | Subdivision of Each Block | 412 |
| 6.6.3 | Connection of Individual Blocks | 413 |
| 6.7 | Bandwidth Reduction | 420 |
| 6.8 | Higher Order Elements | 424 |
| 6.8.1 | Pascal Triangle | 425 |
| 6.8.2 | Local Coordinates | 426 |
| 6.8.3 | Shape Functions | 427 |
| 6.8.4 | Fundamental Matrices | 430 |
| 6.9 | Three-Dimensional Elements | 439 |
| 6.10 | Finite Element Methods for Exterior Problems | 444 |
| 6.10.1 | Infinite Element Method | 444 |
| 6.10.2 | Boundary Element Method | 446 |
| 6.10.3 | Absorbing Boundary Conditions | 446 |
| 6.11 | Concluding Remarks | 448 |
| | References | 449 |
| | Problems | 458 |
| 7 | Transmission-line-matrix Method | 467 |
| 7.1 | Introduction | 467 |
| 7.2 | Transmission-line Equations | 469 |
| 7.3 | Solution of Diffusion Equation | 473 |
| 7.4 | Solution of Wave Equations | 477 |
| 7.4.1 | Equivalence Between Network and Field Parameters | 477 |
| 7.4.2 | Dispersion Relation of Propagation Velocity | 481 |
| 7.4.3 | Scattering Matrix | 483 |
| 7.4.4 | Boundary Representation | 486 |
| 7.4.5 | Computation of Fields and Frequency Response | 487 |

| | | |
|----------|--|------------|
| 7.4.6 | Output Response and Accuracy of Results | 487 |
| 7.5 | Inhomogeneous and Lossy Media in TLM | 493 |
| 7.5.1 | General Two-Dimensional Shunt Node | 494 |
| 7.5.2 | Scattering Matrix | 496 |
| 7.5.3 | Representation of Lossy Boundaries | 497 |
| 7.6 | Three-Dimensional TLM Mesh | 499 |
| 7.6.1 | Series Nodes | 499 |
| 7.6.2 | Three-Dimensional Node | 504 |
| 7.6.3 | Boundary Conditions | 507 |
| 7.7 | Error Sources and Correction | 517 |
| 7.7.1 | Truncation Error | 518 |
| 7.7.2 | Coarseness Error | 518 |
| 7.7.3 | Velocity Error | 519 |
| 7.7.4 | Misalignment Error | 519 |
| 7.8 | Absorbing Boundary Conditions | 519 |
| 7.9 | Concluding Remarks | 521 |
| | References | 523 |
| | Problems | 529 |
| 8 | Monte Carlo Methods | 537 |
| 8.1 | Introduction | 537 |
| 8.2 | Generation of Random Numbers and Variables | 538 |
| 8.3 | Evaluation of Error | 542 |
| 8.4 | Numerical Integration | 546 |
| 8.4.1 | Crude Monte Carlo Integration | 546 |
| 8.4.2 | Monte Carlo Integration with Antithetic Variates | 548 |
| 8.4.3 | Improper Integrals | 549 |
| 8.5 | Solution of Potential Problems | 550 |
| 8.5.1 | Fixed Random Walk | 552 |
| 8.5.2 | Floating Random Walk | 557 |
| 8.5.3 | Exodus Method | 559 |
| 8.6 | Regional Monte Carlo Methods | 574 |
| 8.7 | Concluding Remarks | 581 |
| | References | 582 |
| | Problems | 588 |
| 9 | Method of Lines | 597 |
| 9.1 | Introduction | 597 |
| 9.2 | Solution of Laplace's Equation | 598 |
| 9.2.1 | Rectangular Coordinates | 598 |
| 9.2.2 | Cylindrical Coordinates | 605 |
| 9.3 | Solution of Wave Equation | 609 |
| 9.3.1 | Planar Microstrip Structures | 612 |
| 9.3.2 | Cylindrical Microstrip Structures | 619 |
| 9.4 | Time-Domain Solution | 627 |

| | | |
|----------|---|------------|
| 9.5 | Concluding Remarks | 629 |
| | References | 629 |
| | Problems | 635 |
| A | Vector Relations | 639 |
| A.1 | Vector Identities | 639 |
| A.2 | Vector Theorems | 639 |
| A.3 | Orthogonal Coordinates | 640 |
| B | Solving Electromagnetic Problems Using C++ | 643 |
| B.1 | Introduction | 643 |
| B.2 | A Brief Description of C++ | 643 |
| B.3 | Object-Oriented | 661 |
| B.4 | C++ Object-Oriented Language Features | 665 |
| B.5 | A Final Note | 674 |
| | References | 675 |
| C | Numerical Techniques in C++ | 677 |
| D | Solution of Simultaneous Equations | 701 |
| D.1 | Elimination Methods | 701 |
| | D.1.1 Gauss's Method | 702 |
| | D.1.2 Cholesky's Method | 703 |
| D.2 | Iterative Methods | 706 |
| | D.2.1 Jacobi's Method | 706 |
| | D.2.2 Gauss-Seidel Method | 708 |
| | D.2.3 Relaxation Method | 708 |
| | D.2.4 Gradient Methods | 710 |
| D.3 | Matrix Inversion | 713 |
| D.4 | Eigenvalue Problems | 714 |
| | D.4.1 Iteration (or Power) Method | 716 |
| | D.4.2 Jacobi's Method | 717 |
| E | Answers to Odd-Numbered Problems | 725 |
| | Index | 741 |