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# **State-space Realisations of Linear 2-D Systems with Extensions to the General $nD$ ( $n > 2$ ) Case**



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# Table of Contents

|   |    |
|---|----|
| <b>Notation</b>   | xi |
| <b>1. Introduction</b>  | 1  |
| <b>2. Preliminaries</b>   | 5  |
| 2.1 State-space Models  | 5  |
| 2.2 Transfer Function Matrices and Polynomial Description   | 9  |
| 2.3 The Gałkowski Approach – the SISO Case  | 12 |
| 2.4 Singularity   | 13 |
| <b>3. The Elementary Operation Algorithm for Polynomial Matrices</b>  | 17 |
| 3.1 Basics  | 17 |
| 3.2 Analysis of the Elementary Operation Algorithm  | 28 |
| 3.2.1 The EOA and Polynomial Matrix Description   | 28 |
| 3.2.2 The EOA and Similarity  | 30 |
| 3.3 The Elementary Operation Algorithm and Singularity  | 33 |
| 3.4 Initial Representations for the Elementary Operation Algorithm  | 39 |
| 3.5 Relationships between the Roesser and the Fornasini-Marchesini Models   | 56 |
| <b>4. 2D State-space Realizations and the Elementary Operation Algorithm – the Single Input - Single Output (SISO) Case</b> | 65 |

|  |            |
|--|------------|
| 4.1 Basics .....   | 65         |
| 4.2 Links between the Roesser and Fornasini-Marchesini Models by the EOA                                 | 84         |
| <b>5. MIMO Systems – the 1D Case .....</b>   | <b>91</b>  |
| 5.1 Multi-input Single-output Systems .....  | 93         |
| 5.2 Single-input Multi-output Systems .....  | 95         |
| 5.3 MISO and SIMO Cases–the Dual Approach .....  | 101        |
| 5.4 General MIMO Case .....  | 106        |
| 5.5 Further Analysis .....   | 113        |
| 5.4 An Algebraic Characterization .....  | 122        |
| <b>6. Multiple-input, Multiple-output (MIMO) Systems – the 2D Case .....</b>                             | <b>125</b> |
| 6.1 The General Approach .....   | 125        |
| 6.1.1 Column Least Common Multiplicity Based Approach .....  | 126        |
| 6.1.2 Row Least Common Multiplicity Based Approach .....   | 132        |
| 6.1.3 A Standard (Nonsingular) Realization Achieved by Direct Application<br>of the EOA .....            | 136        |
| 6.2 Algebraic Justification for the MIMO Case .....  | 143        |
| 6.3 Variable Transformations for the Elementary Operation Algorithm .....                                | 146        |
| 6.3.1 Variable Inversion .....   | 146        |
| 6.3.2 Generalised Bilinear Transform .....   | 152        |
| <b>7. The Elementary Operation Approach Extensions .....</b>   | <b>167</b> |
| 7.1 The $nD$ Elementary Operation Algorithm .....  | 167        |
| 7.2 Applications to Repetitive Processes .....   | 177        |
| 7.3 The Elementary Operation Algorithm and Laurent Polynomials .....                                     | 182        |
| <b>8. State-Space Realizations for 2D/<math>nD</math> Systems Revisited –<br/>Relations to EOA .....</b> | <b>187</b> |
| 8.1 Overview .....   | 187        |
| 8.2 <i>A priori</i> Non-minimal Realization .....  | 198        |
| 8.3 Multi-way Companion Matrices .....   | 204        |
| 8.3.1 The History of Multi-way Matrices .....  | 204        |

|  |            |
|--|------------|
| 8.3.2 Multi-way Matrices - Mathematical Basics .....                       | 205        |
| 8.3.3 Three-way Companion Matrix of a Trivariate Polynomial .....          | 207        |
| 8.4 Minimal Realization for Multi-linear n-variate Transfer Functions..... | 209        |
| <b>9. Conclusions and Further Work.....</b>                                | <b>219</b> |
| <b>References .....</b>  | <b>223</b> |
| <b>Index.....</b>  | <b>229</b> |