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Imperfect Bifurcation in Structures and Materials

Engineering Use of Group-Theoretic
Bifurcation Theory

With 194 Illustrations



Springer

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