

Wolfgang Ebeling

Lattices and Codes

A Course Partially Based on Lectures by F. Hirzebruch

2nd Revised Edition



Contents

1	Lattices and Codes	1
1.1	Lattices	1
1.2	Codes	6
1.3	From Codes to Lattices	12
1.4	Root Lattices	16
1.5	Highest Root and Weyl Vector	31
2	Theta Functions and Weight Enumerators	39
2.1	The Theta Function of a Lattice	39
2.2	Modular Forms	40
2.3	The Poisson Summation Formula	44
2.4	Theta Functions as Modular Forms	46
2.5	The Eisenstein Series	48
2.6	The Algebra of Modular Forms	54
2.7	The Weight Enumerator of a Code	62
2.8	The Golay Code and the Leech Lattice	64
2.9	The MacWilliams Identity and Gleason's Theorem	72
2.10	Quadratic Residue Codes	77
3	Even Unimodular Lattices	87
3.1	Theta Functions with Spherical Coefficients	87
3.2	Root Systems in Even Unimodular Lattices	99
3.3	Overlattices and Codes	102
3.4	The Classification of Even Unimodular Lattices of Dimension 24	106
4	The Leech Lattice	109
4.1	The Uniqueness of the Leech Lattice	109
4.2	The Sphere Covering Determined by the Leech Lattice	111
4.3	Twenty-Three Constructions of the Leech Lattice	116
4.4	Embedding the Leech Lattice in a Hyperbolic Lattice	119

4.5	Automorphism Groups	131
5	Lattices over Integers of Number Fields and Self-Dual Codes	135
5.1	Lattices over Integers of Cyclotomic Fields	135
5.2	Construction of Lattices from Codes over \mathbb{F}_p	141
5.3	Theta Functions over Number Fields	146
5.4	The Case $p = 3$: Ternary Codes	150
5.5	The Equation of the Tetrahedron and the Cube	158
5.6	The Case $p = 5$: the Icosahedral Group	161
5.7	Theta Functions as Hilbert Modular Forms (by N.-P. Skoruppa)	166
	Bibliography	175
	Index	183