

MATHEMATICAL METHODS USING *MATHEMATICA*[®]

For Students of Physics
and Related Fields

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With 93 Illustrations and a CD-ROM



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