VERIFICATION of **COMPUTER CODES** in COMPUTATIONAL SCIENCE and **ENCINEERINC Patrick Knupp** Kambiz Salari



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