

Coding Theory and Number Theory

by

Toyokazu Hiramatsu

*Hosei University,
Tokyo, Japan*

and

Günter Köhler

*Würzburg University,
Würzburg, Germany*



KLUWER ACADEMIC PUBLISHERS

DORDRECHT / BOSTON / LONDON

Contents

Preface	ix
1. LINEAR CODES	1
1 Coding Theory	1
2 Linear codes	4
3 Cyclic codes	7
4 Finite fields	13
5 BCH codes	14
2. DIOPHANTINE EQUATIONS AND CYCLIC CODES	17
1 Diagonal equations over finite fields	17
2 The number of solutions and weight distributions of cyclic codes	18
3. ELLIPTIC CURVES, HECKE OPERATORS AND WEIGHT DISTRIBUTION OF CODES	23
1 Elliptic curves over finite fields	23
1.1 The group law	24
1.2 Hasse's theorem	27
2 Modular forms and Hecke operators	29
2.1 $SL_2(\mathbf{Z})$ and its congruence subgroups	29
2.2 The upper half-plane	30
2.3 Modular forms and cusp forms	30
2.4 Hecke operators	32
3 Weight distribution of linear codes	35
3.1 The MacWilliams identities and Delsarte's theorem	35
3.2 The weight distribution of the Melas codes	39

4.	ALGEBRAIC-GEOMETRIC CODES AND MODULAR CURVE CODES	49
1	Classical Goppa codes	49
1.1	The basic idea	49
1.2	Basic properties of $\Gamma(L, g)$	51
2	Algebraic curves	52
2.1	Affine varieties and projective varieties	52
2.2	Divisors of algebraic curves	55
3	The zeta functions of curves and rational points	58
3.1	Basic properties of the zeta functions	58
3.2	Maximum number of rational points	61
4	Algebraic-geometric codes	62
4.1	Algebraic-geometric codes of the first kind	62
4.2	Algebraic-geometric codes of the second kind	65
5	Modular curves and codes	66
5.1	Modular curves and their reduction modulo p	67
5.2	Proof of Theorem 4.8	69
5.3	Asymptotic bounds	72
5.	THETA FUNCTIONS AND SELF-DUAL CODES	77
1	Lattices and codes	77
1.1	Lattices	77
1.2	Constructing lattices from binary codes	81
1.3	Examples	83
1.4	The lattices associated to a code and the dual code	91
2	Theta functions and weight distributions	92
2.1	The theta function of a lattice	92
2.2	The MacWilliams identity	101
3	Doubly-even self-dual codes	105
3.1	The weight enumerator of doubly-even self-dual codes	105
3.2	The extended Golay code	108
3.3	The Leech lattice	112
	Appendix	116
	The Kloosterman Codes and Distribution of the Weights	117
1	Introduction	117

<i>Contents</i>	vii
2 Melas code and Kloosterman sums	119
3 Hyper-Kloosterman code	122
4 Quasi-cyclic property	126
5 Weight distribution	128
6 Minimum distance of $C_m(q)$	133
7 A divisibility theorem for Hamming weights	136
References	141
Index	145