3-MANIFOLDS

JOHN HEMPEL

AMS CHELSEA PUBLISHING American Mathematical Society • Providence, Rhode Island

CONTENTS

PREFACE		vii
1	PRELIMINARIES Definitions Basic Theorems Regular Neighborhoods General Position	3 4 6 7 8
2	HEEGAARD SPLITTINGS Cubes with Handles Splittings and Diagrams Genus One Splittings	14 15 17 20
3	CONNECTED SUMS Primes Existence of Factorizations Uniqueness of Factorizations	24 27 29 32
4	THE LOOP AND SPHERE THEOREMS Double Curve Surgery Proof of the Loop Theorem Proof of the Sphere Theorem The Projective Plane Theorem	39 41 47 50 54
5	FREE GROUPS	56
6	INCOMPRESSIBLE SURFACES	58
7	KNESER'S CONJECTURE ON FREE PRODUCTS	66
8	FINITELY GENERATED SUBGROUPS	69
9	MORE ON CONNECTED SUMS: FINITE AND ABELIAN SUBGROUPS Group Homology Finite Groups: The Nonorientable Case Subgroups with Higher Homology Abelian Groups	75 75 76 80 84
10	I-BUNDLES Products Twisted Bundles Surface Subgroups of Finite Index	88 89 91 98

11	GROUP EXTENSIONS AND FIBRATIONS Algebraic Preliminaries Bundles Proof of Theorem 11.1	100 101 103 110
12	SEIFERT FIBERED SPACES Fuchsian Groups Bundles with Period Structure Groups Cyclic Normal Subgroups Centers Cyclic Actions on $S^1 \times S^1 \times S^1$	115 118 121 125 131 132
13	CLASSIFICATION OF P ² -IRREDUCIBLE, SUFFICIENTLY LARGE 3-MANIFOLDS The Analogue for Surfaces Hierarchies Classification Theorems Peripheral Systems Remarks and Examples	136 137 140 143 149 150
14	SOME APPROACHES TO THE POINCARE CONJECTURE Contractible Open 3-Manifolds A Characterization of S ³ Splitting Homomorphisms The Mapping Class Group Involutions on Homotopy 3-Spheres	154 155 157 158 162 164
15	OPEN PROBLEMS The Fundamental Groups Peripheral Systems Hopficity Residual Finiteness	169 169 172 175 176
RE	FERENCES	185
INDEX		192
SYN	MBOLS AND NOTATION	194

-

CONTENTS

xii