

George C. Hsiao Wolfgang L. Wendland

Boundary Integral Equations

 Springer

Table of Contents

Preface	VII
1. Introduction	1
1.1 The Green Representation Formula	1
1.2 Boundary Potentials and Calderón's Projector	3
1.3 Boundary Integral Equations.....	10
1.3.1 The Dirichlet Problem	11
1.3.2 The Neumann Problem	12
1.4 Exterior Problems	13
1.4.1 The Exterior Dirichlet Problem	13
1.4.2 The Exterior Neumann Problem	15
1.5 Remarks	19
2. Boundary Integral Equations	25
2.1 The Helmholtz Equation	25
2.1.1 Low Frequency Behaviour	31
2.2 The Lamé System	45
2.2.1 The Interior Displacement Problem	47
2.2.2 The Interior Traction Problem	55
2.2.3 Some Exterior Fundamental Problems	56
2.2.4 The Incompressible Material	61
2.3 The Stokes Equations	62
2.3.1 Hydrodynamic Potentials.....	65
2.3.2 The Stokes Boundary Value Problems.....	66
2.3.3 The Incompressible Material — Revisited	75
2.4 The Biharmonic Equation	79
2.4.1 Calderón's Projector.....	83
2.4.2 Boundary Value Problems and Boundary Integral Equations	85
2.5 Remarks	91
3. Representation Formulae	95
3.1 Classical Function Spaces and Distributions.....	95
3.2 Hadamard's Finite Part Integrals	101

3.3	Local Coordinates	108
3.4	Short Excursion to Elementary Differential Geometry	111
3.4.1	Second Order Differential Operators in Divergence Form	119
3.5	Distributional Derivatives and Abstract Green's Second Formula	126
3.6	The Green Representation Formula	130
3.7	Green's Representation Formulae in Local Coordinates	135
3.8	Multilayer Potentials	139
3.9	Direct Boundary Integral Equations	145
3.9.1	Boundary Value Problems	145
3.9.2	Transmission Problems	155
3.10	Remarks	157
4.	Sobolev Spaces	159
4.1	The Spaces $H^s(\Omega)$	159
4.2	The Trace Spaces $H^s(\Gamma)$	169
4.2.1	Trace Spaces for Periodic Functions on a Smooth Curve in \mathbb{R}^2	181
4.2.2	Trace Spaces on Curved Polygons in \mathbb{R}^2	185
4.3	The Trace Spaces on an Open Surface	189
4.4	Weighted Sobolev Spaces	191
5.	Variational Formulations	195
5.1	Partial Differential Equations of Second Order	195
5.1.1	Interior Problems	199
5.1.2	Exterior Problems	204
5.1.3	Transmission Problems	215
5.2	Abstract Existence Theorems for Variational Problems	218
5.2.1	The Lax–Milgram Theorem	219
5.3	The Fredholm–Nikolski Theorems	226
5.3.1	Fredholm's Alternative	226
5.3.2	The Riesz–Schauder and the Nikolski Theorems	235
5.3.3	Fredholm's Alternative for Sesquilinear Forms	240
5.3.4	Fredholm Operators	241
5.4	Gårding's Inequality for Boundary Value Problems	243
5.4.1	Gårding's Inequality for Second Order Strongly Elliptic Equations in Ω	243
5.4.2	The Stokes System	250
5.4.3	Gårding's Inequality for Exterior Second Order Problems	254
5.4.4	Gårding's Inequality for Second Order Transmission Problems	259
5.5	Existence of Solutions to Boundary Value Problems	259
5.5.1	Interior Boundary Value Problems	260

5.5.2	Exterior Boundary Value Problems	264
5.5.3	Transmission Problems	264
5.6	Solution of Integral Equations via Boundary Value Problems .	265
5.6.1	The Generalized Representation Formula for Second Order Systems	265
5.6.2	Continuity of Some Boundary Integral Operators	267
5.6.3	Continuity Based on Finite Regions	270
5.6.4	Continuity of Hydrodynamic Potentials	272
5.6.5	The Equivalence Between Boundary Value Problems and Integral Equations	274
5.6.6	Variational Formulation of Direct Boundary Integral Equations	277
5.6.7	Positivity and Contraction of Boundary Integral Operators	287
5.6.8	The Solvability of Direct Boundary Integral Equations	291
5.6.9	Positivity of the Boundary Integral Operators of the Stokes System	292
5.7	Partial Differential Equations of Higher Order	293
5.8	Remarks	299
5.8.1	Assumptions on Γ	299
5.8.2	Higher Regularity of Solutions	299
5.8.3	Mixed Boundary Conditions and Crack Problem	300
6.	Introduction to Pseudodifferential Operators	303
6.1	Basic Theory of Pseudodifferential Operators	303
6.2	Elliptic Pseudodifferential Operators on $\Omega \subset \mathbb{R}^n$	326
6.2.1	Systems of Pseudodifferential Operators	328
6.2.2	Parametrix and Fundamental Solution	331
6.2.3	Levi Functions for Scalar Elliptic Equations	334
6.2.4	Levi Functions for Elliptic Systems	341
6.2.5	Strong Ellipticity and Gårding's Inequality	343
6.3	Review on Fundamental Solutions	346
6.3.1	Local Fundamental Solutions	347
6.3.2	Fundamental Solutions in \mathbb{R}^n for Operators with Constant Coefficients	348
6.3.3	Existing Fundamental Solutions in Applications	352
7.	Pseudodifferential Operators as Integral Operators	353
7.1	Pseudohomogeneous Kernels	353
7.1.1	Integral Operators as Pseudodifferential Operators of Negative Order	356
7.1.2	Non-Negative Order Pseudodifferential Operators as Hadamard Finite Part Integral Operators	380

7.1.3	Parity Conditions	389
7.1.4	A Summary of the Relations between Kernels and Symbols	392
7.2	Coordinate Changes and Pseudohomogeneous Kernels	394
7.2.1	The Transformation of General Hadamard Finite Part Integral Operators under Change of Coordinates	397
7.2.2	The Class of Invariant Hadamard Finite Part Integral Operators under Change of Coordinates	404
8.	Pseudodifferential and Boundary Integral Operators	413
8.1	Pseudodifferential Operators on Boundary Manifolds	414
8.1.1	Ellipticity on Boundary Manifolds	418
8.1.2	Schwartz Kernels on Boundary Manifolds	420
8.2	Boundary Operators Generated by Domain Pseudodifferential Operators	421
8.3	Surface Potentials on the Plane \mathbb{R}^{n-1}	423
8.4	Pseudodifferential Operators with Symbols of Rational Type .	446
8.5	Surface Potentials on the Boundary Manifold Γ	467
8.6	Volume Potentials	476
8.7	Strong Ellipticity and Fredholm Properties	479
8.8	Strong Ellipticity of Boundary Value Problems and Associated Boundary Integral Equations	485
8.8.1	The Boundary Value and Transmission Problems	485
8.8.2	The Associated Boundary Integral Equations of the First Kind	488
8.8.3	The Transmission Problem and Gårding's inequality ..	489
8.9	Remarks	491
9.	Integral Equations on $\Gamma \subset \mathbb{R}^3$ Recast as Pseudodifferential Equations	493
9.1	Newton Potential Operators for Elliptic Partial Differential Equations and Systems	499
9.1.1	Generalized Newton Potentials for the Helmholtz Equation	502
9.1.2	The Newton Potential for the Lamé System	505
9.1.3	The Newton Potential for the Stokes System	506
9.2	Surface Potentials for Second Order Equations	507
9.2.1	Strongly Elliptic Differential Equations	510
9.2.2	Surface Potentials for the Helmholtz Equation	514
9.2.3	Surface Potentials for the Lamé System	519
9.2.4	Surface Potentials for the Stokes System	524
9.3	Invariance of Boundary Pseudodifferential Operators	524
9.3.1	The Hypersingular Boundary Integral Operators for the Helmholtz Equation	525

9.3.2	The Hypersingular Operator for the Lamé System	531
9.3.3	The Hypersingular Operator for the Stokes System . . .	535
9.4	Derivatives of Boundary Potentials	535
9.4.1	Derivatives of the Solution to the Helmholtz Equation	541
9.4.2	Computation of Stress and Strain on the Boundary for the Lamé System	543
9.5	Remarks	547
10.	Boundary Integral Equations on Curves in \mathbb{R}^2	549
10.1	Fourier Series Representation of the Basic Operators	550
10.2	The Fourier Series Representation of Periodic Operators $A \in \mathcal{L}_{cl}^m(\Gamma)$	556
10.3	Ellipticity Conditions for Periodic Operators on Γ	562
10.3.1	Scalar Equations	563
10.3.2	Systems of Equations	568
10.3.3	Multiply Connected Domains	572
10.4	Fourier Series Representation of some Particular Operators . .	574
10.4.1	The Helmholtz Equation	574
10.4.2	The Lamé System	578
10.4.3	The Stokes System	581
10.4.4	The Biharmonic Equation	582
10.5	Remarks	591
A.	Differential Operators in Local Coordinates with Minimal Differentiability	593
	References	599
	Index	613