

# Topics in Structural Graph Theory

Edited by

LOWELL W. BEINEKE  
*Indiana University–Purdue University  
Fort Wayne*

ROBIN J. WILSON  
*The Open University  
and Pembroke College, Oxford University*

Academic Consultant

ORTRUD R. OELLERMANN  
*University of Winnipeg*



**CAMBRIDGE**  
UNIVERSITY PRESS

# Contents

---

<i>Foreword by Ortrud R. Oellermann</i>	<i>page xi</i>
<i>Preface</i>	<i>xiii</i>
<b>Preliminaries</b>	<b>1</b>
LOWELL W. BEINEKE and ROBIN J. WILSON	
1. Graph theory	1
2. Connectivity	8
3. Flows in networks	10
<b>1 Menger's theorem</b>	<b>13</b>
ORTRUD R. OELLERMANN	
1. Introduction	13
2. Vertex-connectivity	14
3. Edge-connectivity	18
4. Mixed connectivity	19
5. Average connectivity	22
6. Menger results for paths of bounded length	28
7. Connectivity of sets	30
8. Connecting with trees	32
<b>2 Maximally connected graphs</b>	<b>40</b>
DIRK MEIERLING and LUTZ VOLKMANN	
1. Introduction	40
2. Maximally edge-connected graphs	41
3. Maximally edge-connected digraphs	46
4. Maximally locally edge-connected graphs and digraphs	48
5. Maximally connected and maximally locally connected graphs and digraphs	50
6. Restricted edge-connectivity	54
7. Conditional vertex-connectivity and edge-connectivity	58

<b>3</b>	<b>Minimal connectivity</b>	<b>71</b>
	MATTHIAS KRIESELL	
	1. Introduction	71
	2. Edge-deletion	73
	3. Vertex-deletion	74
	4. Edge-contraction	79
	5. Generalized criticality	81
	6. Reduction methods	82
	7. Subgraph-deletion	88
	8. Partitions under connectivity constraints	91
	9. Line graphs	94
<b>4</b>	<b>Contractions of <math>k</math>-connected graphs</b>	<b>100</b>
	KIYOSHI ANDO	
	1. Introduction	100
	2. Contractible edges in 3-connected graphs	101
	3. Contractible edges in 4-connected graphs	102
	4. Contractible edges in $k$ -connected graphs	103
	5. Contraction-critical 5-connected graphs	106
	6. Local structure and contractible edges	109
	7. Concluding remarks	111
<b>5</b>	<b>Connectivity and cycles</b>	<b>114</b>
	R. J. FAUDREE	
	1. Introduction	114
	2. Generalizations of classical results	115
	3. Relative lengths of paths and cycles	117
	4. Regular graphs	119
	5. Bipartite graphs	122
	6. Claw-free graphs	123
	7. Planar graphs	128
	8. The Chvátal–Erdős condition	131
	9. Ordered graphs	132
	10. Numbers of cycles	134
<b>6</b>	<b><math>H</math>-linked graphs</b>	<b>141</b>
	MICHAEL FERRARA <sup>1</sup> and RONALD J. GOULD	
	1. Introduction	141
	2. $k$ -linked graphs	143
	3. Weak linkage	149
	4. Digraphs	150
	5. Modulo and parity linkage	152

6.	Disjoint connected subgraphs	154
7.	The disjoint paths problem	154
8.	$H$ -linked graphs	155
9.	$H$ -extendible graphs	159
<b>7</b>	<b>Tree-width and graph minors</b>	<b>165</b>
	DIETER RAUTENBACH and BRUCE REED	
1.	Introduction	165
2.	Subtree intersection representation	166
3.	Tree decomposition and tree-width	168
4.	Tree decompositions decompose	173
5.	Excluding planar minors	174
6.	Wagner's conjecture	175
7.	The dual of tree-width	176
8.	A canonical tree decomposition	178
9.	Wagner's conjecture for arbitrary graphs	180
10.	Efficient characterization of $H$ -minor-free graphs	181
<b>8</b>	<b>Toughness and binding numbers</b>	<b>185</b>
	IAN ANDERSON	
1.	Introduction	185
2.	Toughness and connectivity	187
3.	Toughness and cycles	188
4.	Toughness and $k$ -factors	191
5.	Binding number	194
6.	Binding number and $k$ -factors	196
7.	Binding numbers and cycles	198
8.	Other measures of vulnerability	198
<b>9</b>	<b>Graph fragmentability</b>	<b>203</b>
	KEITH EDWARDS and GRAHAM FARR	
1.	Introduction	203
2.	Values and bounds for fragmentability	206
3.	Reduction and separation	207
4.	Bounded degree classes	208
5.	Planarization	210
6.	Applications	214
7.	Monochromatic components	215
8.	Open problems	216
<b>10</b>	<b>The phase transition in random graphs</b>	<b>219</b>
	BÉLA BOLLOBÁS and OLIVER RIORDAN	

1.	Introduction	219
2.	The Erdős–Rényi theorem: the double jump	223
3.	Correction: no double jump	225
4.	The phase transition – simple results	227
5.	Exploring components	238
6.	The phase transition – finer results	240
7.	The young giant	243
8.	Final words	247
<b>11</b>	<b>Network reliability and synthesis</b>	<b>251</b>
	F. T. BOESCH, A. SATYANARAYANA and C. L. SUFFEL	
1.	Introduction	251
2.	Domination in digraphs	252
3.	Coherent systems and domination in graphs	255
4.	Computational complexity of reliability	260
5.	Synthesis of reliable networks	260
6.	Other measures of vulnerability	263
<b>12</b>	<b>Connectivity algorithms</b>	<b>268</b>
	ABDOL-HOSSEIN ESFAHANIAN	
1.	Introduction	268
2.	Computing the edge-connectivity	269
3.	Computing the arc-connectivity	274
4.	Computing the vertex-connectivity	275
5.	Concluding remarks	279
<b>13</b>	<b>Using graphs to find the best block designs</b>	<b>282</b>
	R. A. BAILEY and PETER J. CAMERON	
1.	What makes a block design good?	283
2.	Graphs from block designs	284
3.	Statistical issues	288
4.	Highly patterned block designs	292
5.	D-optimality	293
6.	A-optimality	294
7.	E-optimality	302
8.	Some history	304
9.	Block size 2	306
10.	Low average replication	311
11.	Further reading	314
	<i>Notes on contributors</i>	<b>318</b>
	<i>Index</i>	<b>323</b>