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Mathematical Control Theory

Deterministic Finite Dimensional Systems

Second Edition

With 26 Illustrations

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Contents

Series Preface v

Preface to the Second Edition vii

Preface to the First Edition ix

1 Introduction 1
- 1.1 What Is Mathematical Control Theory? . . . 1
- 1.2 Proportional-Derivative Control . . . 2
- 1.3 Digital Control . . . 6
- 1.4 Feedback Versus Precomputed Control . . . 9
- 1.5 State-Space and Spectrum Assignment . . . 11
- 1.6 Outputs and Dynamic Feedback . . . 16
- 1.7 Dealing with Nonlinearity . . . 20
- 1.8 A Brief Historical Background . . . 22
- 1.9 Some Topics Not Covered . . . 23

2 Systems 25
- 2.1 Basic Definitions . . . 25
- 2.2 I/O Behaviors . . . 30
- 2.3 Discrete-Time . . . 32
- 2.4 Linear Discrete-Time Systems . . . 36
- 2.5 Smooth Discrete-Time Systems . . . 39
- 2.6 Continuous-Time . . . 41
- 2.7 Linear Continuous-Time Systems . . . 46
- 2.8 Linearizations Compute Differentials . . . 53
- 2.9 More on Differentiability* . . . 64
- 2.10 Sampling . . . 72
- 2.11 Volterra Expansions* . . . 73
- 2.12 Notes and Comments . . . 76

*Can be skipped with no loss of continuity.

3 Reachability and Controllability **81**
3.1 Basic Reachability Notions 81
3.2 Time-Invariant Systems 84
3.3 Controllable Pairs of Matrices 92
3.4 Controllability Under Sampling 99
3.5 More on Linear Controllability 104
3.6 Bounded Controls* 117
3.7 First-Order Local Controllability 122
3.8 Controllability of Recurrent Nets* 128
3.9 Piecewise Constant Controls 136
3.10 Notes and Comments 137

4 Nonlinear Controllability **141**
4.1 Lie Brackets 141
4.2 Lie Algebras and Flows 147
4.3 Accessibility Rank Condition 154
4.4 Ad, Distributions, and Frobenius' Theorem 164
4.5 Necessity of Accessibility Rank Condition 177
4.6 Additional Problems 179
4.7 Notes and Comments 181

5 Feedback and Stabilization **183**
5.1 Constant Linear Feedback 183
5.2 Feedback Equivalence* 189
5.3 Feedback Linearization* 197
5.4 Disturbance Rejection and Invariance* 207
5.5 Stability and Other Asymptotic Notions 211
5.6 Unstable and Stable Modes* 215
5.7 Lyapunov and Control-Lyapunov Functions 218
5.8 Linearization Principle for Stability 233
5.9 Introduction to Nonlinear Stabilization* 239
5.10 Notes and Comments 256

6 Outputs **261**
6.1 Basic Observability Notions 261
6.2 Time-Invariant Systems 268
6.3 Continuous-Time Linear Systems 276
6.4 Linearization Principle for Observability 280
6.5 Realization Theory for Linear Systems 283
6.6 Recursion and Partial Realization 290
6.7 Rationality and Realizability 297
6.8 Abstract Realization Theory* 303
6.9 Notes and Comments 310

*Can be skipped with no loss of continuity.

7 Observers and Dynamic Feedback **315**
7.1 Observers and Detectability 315
7.2 Dynamic Feedback 321
7.3 External Stability for Linear Systems 326
7.4 Frequency-Domain Considerations 331
7.5 Parametrization of Stabilizers 337
7.6 Notes and Comments 344

8 Optimality: Value Function **347**
8.1 Dynamic Programming* 349
8.2 Linear Systems with Quadratic Cost 363
8.3 Tracking and Kalman Filtering* 371
8.4 Infinite-Time (Steady-State) Problem 380
8.5 Nonlinear Stabilizing Optimal Controls 390
8.6 Notes and Comments 394

9 Optimality: Multipliers **397**
9.1 Review of Smooth Dependence 397
9.2 Unconstrained Controls 399
9.3 Excursion into the Calculus of Variations 409
9.4 Gradient-Based Numerical Methods 415
9.5 Constrained Controls: Minimum Principle 418
9.6 Notes and Comments 421

10 Optimality: Minimum-Time for Linear Systems **423**
10.1 Existence Results 424
10.2 Maximum Principle for Time-Optimality 431
10.3 Applications of the Maximum Principle 436
10.4 Remarks on the Maximum Principle 443
10.5 Additional Exercises 444
10.6 Notes and Comments 446

APPENDIXES

A Linear Algebra **447**
A.1 Operator Norms 447
A.2 Singular Values 448
A.3 Jordan Forms and Matrix Functions 452
A.4 Continuity of Eigenvalues 456

B Differentials **461**
B.1 Finite Dimensional Mappings 461
B.2 Maps Between Normed Spaces 463

*Can be skipped with no loss of continuity.

C Ordinary Differential Equations **467**
C.1 Review of Lebesgue Measure Theory 467
C.2 Initial-Value Problems 473
C.3 Existence and Uniqueness Theorem 474
C.4 Linear Differential Equations 487
C.5 Stability of Linear Equations 491

Bibliography **493**

List of Symbols **519**

Index **523**