

Peter Kennedy

Flow Analysis of Injection Molds



Hanser Publishers, Munich Vienna New York

Hanser/Gardner Publications, Inc., Cincinnati

Contents

Preface	v
List of Figures	xiv
CHAPTER 1	Introduction 1
1.1	The Injection Molding Cycle 1
1.2	The Need for Simulation Software 4
CHAPTER 2	Material Properties 6
2.1	Types of Polymers 6
2.1.1	Amorphous and Semi-crystalline Materials 7
2.2	Viscosity, Shear Rate and Shear Stress 7
2.3	Newtonian and Non-Newtonian Fluids 9
2.4	Viscosity Models for Flow Analysis 9
2.4.1	Power Law Model 10
2.4.2	Moldflow Second Order Model 11
2.4.3	Moldflow Matrix Data Model 12
2.4.4	The Ellis Model 12
2.4.5	The Carreau Model 12
2.4.6	Cross Model 13
2.5	Viscosity Model Requirements 13
2.6	Extrapolation of Viscosity Data 14
2.7	Thermal Properties for Flow Analysis 15
2.7.1	Specific Heat Capacity 16
2.7.2	Thermal Conductivity 17
2.7.3	No-Flow Temperature 19
2.8	Thermodynamic Relationships 25
2.8.1	Expansivity and Compressibility 25
2.8.2	PVT Data for Flow Analysis 28

CHAPTER 3	Stress and Strain in Fluid Mechanics	30
3.1	Stress in Fluids	30
3.1.1	Stress on an External Surface	31
3.1.2	Stress at an Interior Point	32
3.1.3	Hydrostatic Pressure	34
3.2	Rate of Strain Tensor	34
3.3	Compressible and Incompressible Fluids	35
3.4	Newtonian and Non-Newtonian Fluids	35
3.4.1	Shear Rate and Choice of Viscosity Function	37
CHAPTER 4	Governing Equations for Fluid Flow	41
4.1	Mathematical Preliminaries	41
4.1.1	The Material Derivative	42
4.1.2	Gauss's Divergence Theorem	43
4.1.3	Reynold's Transport Theorem	43
4.2	Conservation of Mass	44
4.3	Conservation of Momentum	46
4.4	Conservation of Energy	47
4.4.1	Relating Specific Energy to Temperature	50
4.4.2	The Energy Equation in Terms of Temperature	54
4.5	Boundary Conditions	55
CHAPTER 5	Governing Equations for the Filling Phase	59
5.1	Material Assumptions	60
5.1.1	Melt Compressibility	60
5.1.2	Fluid Model	60
5.1.3	Thermal Properties of the Melt	61
5.1.4	Resulting Equations	62

5.2	Geometric Considerations: The Cavity	62
5.2.1	Dimensional Analysis	62
5.2.2	The Continuity Equation for the Cavity	65
5.2.3	The Momentum Equation for the Cavity	66
5.2.4	The Energy Equation for the Cavity	69
5.2.5	Resulting Equations	71
5.3	Simplification by Mathematical Analysis	72
5.3.1	Integration of the Momentum Equations	72
5.3.2	Integration of the Continuity Equation	79
5.4	Simplification Due to Symmetry	79
5.5	Equations for Runners	81
5.5.1	The Continuity Equation for Runners	82
5.5.2	The Momentum Equations for Runners	83
5.5.3	The Energy Equation for Runners	84
5.5.4	Resulting Equations for Runners	85
5.5.5	Integration of the Momentum Equation for Runners	85
5.5.6	Integration of the Continuity Equation for Runners	88
5.6	Summary	89
CHAPTER 6	Governing Equations for the Packing Phase	91
6.1	The Momentum Equation for the Cavity	91
6.1.1	Effect of Assumptions on Momentum Equation	93
6.2	The Continuity Equation for the Cavity	94
6.3	The Energy Equation for the Cavity	96
6.4	Integration of the Continuity Equation	99
6.5	Packing Equations for Runners	101
6.5.1	Continuity	102
6.5.2	Momentum:	103
6.5.3	Energy	104
6.6	Integration of the Continuity Equation for Runners	104
6.7	Summary	107

CHAPTER 7	Numerical Methods Used in Flow Analysis: The Finite Element Method	109
7.1	Introduction to the Finite Element Method	109
7.1.1	Basic Terminology	110
7.1.2	The Finite Element Approach	110
7.2	The Nature of a Finite Element Solution	114
7.3	Shape Functions	116
7.4	Finding Nodal Values	117
7.4.1	Weighted Residual Methods	117
7.4.2	Galerkin's Method	118
7.5	Constraint Equations	118
7.5.1	Special Case 1: Two Unknowns Equal	121
7.5.2	Special Case 2: One Unknown Constant	123
7.6	A One Dimensional Problem Solved Using the FEM	124
7.6.1	Meshing	125
7.6.2	Derivation of Element Equations	126
7.6.3	Assembly	133
7.6.4	Application of Boundary Conditions	134
7.6.5	Solution of System Equations	136
7.7	Conclusion	137
CHAPTER 8	Numerical Methods Used in Flow Analysis: The Finite Difference Method	138
8.1	Introduction to the Finite Difference Method	138
8.2	A Simple Example	142
8.3	The Diffusion Equation	146
8.3.1	Explicit Methods	146
8.3.2	Implicit Methods	147

CHAPTER 9	Numerical Solution for the Filling Phase	149
9.1	Overview of Solution Process	149
9.2	Finite Element Solution for the Pressure Field	152
9.2.1	Finite Element Formulation for Triangular Elements	152
9.2.2	Finite Element Formulation for Runner Elements	160
9.2.3	Assembly of Element Equations	165
9.2.4	Application of Boundary Conditions	167
9.2.5	Solution of the System Equations	169
9.3	Finite Difference Solution of the Energy Equation for the Cavity	170
9.3.1	Finite Difference Grid Definition	171
9.3.2	Treatment of Convection and Dissipation Terms	172
9.4	Solution of Energy Equation	172
9.4.1	Explicit Finite Difference Method	173
9.4.2	Implicit Finite Difference Method	174
9.4.3	Numerical Solution of the Energy Equation for Runners	175
9.5	Advancement of the Flow Front	176
9.5.1	Nodal Growth of Flow Front	176
9.6	Accounting for Compressibility and Expansivity of the Melt	180
CHAPTER 10	Numerical Solution of the Packing Phase	185
10.1	Overview of the Solution Process	185
10.2	Finite Element Formulation for the Pressure Field in the Cavity	188
10.2.1	Assembly of Element Equations and Solution	192
10.2.2	Finite Element Pressure Solution for Runners	193
10.3	The Use of PVT Data and Thermal Calculations	197

Appendix A	Notation Used in Text	199
A.1	Roman Symbols	199
A.2	Greek Symbols	201
A.3	Mathematical Symbols	202
A.4	Vectors and Tensor Quantities	203
A.5	Results of Vector and Tensor Products	203
Appendix B	Vector and Tensor Operations	204
B.1	Cartesian Coordinates	204
B.1.1	The Gradient Operator	204
B.1.2	Gradient of a Scalar Function	205
B.1.3	Laplacian of a Scalar Function	205
B.1.4	Divergence of a Vector Field	205
B.1.5	Gradient of a Vector Field	205
B.1.6	Dot Product of Tensor and Vector	206
B.1.7	Dyadic Product	206
B.1.8	The Rate of Deformation Tensor	206
B.1.9	Divergence of Product of a Scalar and Tensor	207
B.1.10	The Scalar Product of Two Tensors (Double Dot Product)	208
B.2	Cylindrical Coordinates	208
B.2.1	The Gradient Operator	208
B.2.2	Gradient of a Scalar Function	209
B.2.3	Laplacian of a Scalar Function	209
B.2.4	Divergence of a Vector Field	209
B.2.5	Gradient of a Vector Field	209
B.2.6	Dot Product of Vector and Tensor	210
B.2.7	210
B.2.8	The Rate of Deformation Tensor	210
B.2.9	Divergence of a Tensor	210
B.2.10	The Scalar Product of Two Tensors (Double Dot Product)	211

Appendix C	Dimensional Analysis of Governing Equations	212
C.1	Characteristic Values	212
C.2	Equations for the Filling Phase	213
Appendix D	References	223
Index		227