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Waves Called Solitons

Concepts and Experiments

Second Revised and Enlarged Edition
With 135 Figures



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