

THIRD EDITION

Introduction to Random Signals and Applied Kalman Filtering

WITH MATLAB EXERCISES
AND SOLUTIONS

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