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# Statistical Mechanics of Nonequilibrium Processes

Volume 2: Relaxation  
and Hydrodynamic Processes



Akademie Verlag

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