K.-E. Peiponen E.M. Vartiainen
T. Asakura

Dispersion, Complex Analysis and Optical Spectroscopy

Classical Theory

With 46 Figures

Springer

Contents

1. Classical Dispersion Theory 1
1.1 Equation of Motion 1
1.2 Maxwell's Equations and Medium Properties 3
1.3 Lorentz and Drude Models for Linear Susceptibility 4
1.4 Wave Equation and the Complex Refractive Index 8
1.5 Complex Reflectivity 10

2. Dispersion Relations in Linear Optics 17
2.1 Causality 17
2.2 Hilbert Transforms 21
2.3 Kramers–Kronig Relations in Transmission Spectroscopy 25
2.4 Multiply-Subtractive Kramers–Kronig Relations 29
2.5 Imaginary Angular Frequencies 31
2.6 Kramers–Kronig Relations in Reflection Spectroscopy 34
2.7 Kramers–Kronig Relations for the Effective Optical Constants of Two-Phase Nanocomposites 39
2.8 Dispersion Relations in Magneto-Optics 42

3. Dispersion Relations in Nonlinear Optics 47
3.1 Hyperpolarizability 47
3.2 Anharmonic Lorentz Oscillator Model 49
3.3 Nonlinear Susceptibilities and Causality 52
3.4 Dispersion Relations for Holomorphic Nonlinear Susceptibilities 52
3.5 Examples of Meromorphic Nonlinear and Total Susceptibilities 55
3.6 Dispersion Theory of Meromorphic Susceptibilities 58

4. Conformal Mappings in Analysis of Optical Spectra 61
4.1 Conformal Mappings 61
4.2 Laurent Series Expansion of Complex Refractive Index in Unit Disk 63
4.3 Phase Retrieval Using Laurent Series Expansion of the Complex Derivative of Normal Reflectance 66
4.4 Conformal Mapping in Description of Oblique-Angle Reflectivity 68

4.5 Conformal Mapping of Nonlinear Susceptibilities 69
4.6 Conformal Mapping in Analysis of the Modulus of Nonlinear Susceptibility 73

5. Maximum Entropy Method 79
5.1 Maximum Entropy Model 79
5.2 Phase Retrieval Procedure 85
5.3 Applications in Reflection Spectroscopy 88
5.4 Applications in Nonlinear Optical Spectroscopy 90

6. Sum Rules 97
6.1 f-Sum Rule and Average-Optical Constant Sum Rules 97
6.2 Sum Rules for the Powers of the Complex Refractive Index .. 100
6.3 Sum Rules for the Powers of the Complex Reflectivity 102
6.4 Sum Rules in Magneto-Optics 106
6.5 Sum Rules in Nonlinear Optics 107
6.6 Poles and Zeros of Meromorphic Susceptibility 111

Appendices 113
A Cauchy's Integral Theorem 113
B Theorem of Residues 113
C Jordan's Lemma 114
D Phase Retrieval for Oblique-Angle Reflectance: s-Polarization 115
E Complex Analysis with Several Variables 116
F Surface Integral of Generalized Cauchy Formula 117
G Derivation of (4.40) 119
H Derivation of (5.21) 120
I Superconvergence Theorem [33] 122

References 123

Index 129