

Esko Turunen

Mathematics Behind Fuzzy Logic

Physica-Verlag

A Springer-Verlag Company

Contents

1	Residuated Lattices	1
1.1	Lattices and Equivalence Relations	1
1.2	Lattice Filters	7
1.3	Residuated Lattices	9
1.4	BL-Algebras	16
1.5	Exercises	29
2	MV-Algebras	35
2.1	MV-Algebras and Wajsberg Algebras	35
2.2	Complete MV-Algebras	48
2.3	Pseudo-Boolean Algebras	68
2.4	Exercises	72
3	Fuzzy Propositional Logic	75
3.1	Semantics of Fuzzy Propositional Logic	75
3.2	Axiom System for Fuzzy Logic	85
3.3	Completeness of Fuzzy Logic	96
3.4	Exercises	105
4	Fuzzy Relations	113
4.1	Solvability of Fuzzy Relation Equations	113
4.2	On Fuzzy Similarity Relations	118
4.3	Fuzzy Similarity and Approximate Reasoning	127
4.4	Maximal Similarity and Fuzzy Reasoning	136
4.4.1	Fuzzy Traffic Signals	137
4.4.2	Determining Athlete's Anaerobic Thresholds	140
4.4.3	Classification Tasks	143
4.5	Exercises	145
5	Solutions to Exercises	149
5.1	Exercises from Section 1.1	149
5.2	Exercises from Section 1.2	152
5.3	Exercises from Section 1.3	152

5.3 Exercises from Section 1.3	152
5.4 Exercises from Section 1.4	157
5.5 Exercises from Section 2.1	160
5.6 Exercises from Section 2.2	164
5.7 Exercises from Section 2.3	168
5.8 Exercises from Section 3.1	169
5.9 Exercises from Section 3.2	173
5.10 Exercises from Section 3.3	175
5.11 Exercises from Section 4.1	179
5.12 Exercises from Section 4.2	181
5.13 Exercises from Section 4.3	181
5.14 Exercises from Section 4.4	182
Bibliography	185
Index	189