# Hybrid Graph Theory and Network Analysis 

Ladislav Novak
University of Novi Sad

Alan Gibbons<br>University of Liverpool

## Contents

Preface ..... ix
1 Two Dual Structures of a Graph ..... 1
1.1 Basic concepts of graphs ..... 1
1.2 Cuts and circs ..... 5
1.3 Cut and circ spaces ..... 9
1.4 Relationships between cut and circ spaces ..... 12
1.5 Edge-separators and connectivity ..... 16
1.6 Equivalence relations among graphs ..... 19
1.7 Directed graphs ..... 22
1.8 Networks and multiports ..... 25
1.9 Kirchhoff's laws ..... 33
1.10 Bibliographic notes ..... 38
2 Independence Structures ..... 39
2.1 The graphoidal point of view ..... 39
2.2 Independent collections of circs and cuts ..... 42
2.3 Maximal circless and cutless sets ..... 48
2.4 Circ and cut vector spaces ..... 55
2.5 Binary graphoids and their representations ..... 62
2.6 Orientable binary graphoids and Kirchhoff's laws ..... 71
2.7 Mesh and nodal analysis ..... 76
2.8 Bibliographic notes ..... 79
3 Basoids ..... 80
3.1 Preliminaries ..... 80
3.2 Basoids of graphs ..... 81
3.3 Transitions from one basoid to another ..... 87
3.4 Minor with respect to a basoid ..... 91
3.5 Principal sequence ..... 94
3.6 Principal minor and principal partition ..... 99
3.7 Hybrid rank and basic pairs of subsets ..... 102
3.8 Hybrid analysis of networks ..... 105
3.9 Procedure for finding an optimal basic pair ..... 109
3.10 Bibliographic notes ..... 113
4 Pairs of Trees ..... 115
4.1 Diameter of a tree ..... 115
4.2 Perfect pairs of trees ..... 119
4.3 Basoids and perfect pairs of trees ..... 126
4.4 Superperfect pairs of trees ..... 130
4.5 Unique solvability of affine networks ..... 134
4.6 Bibliographic notes ..... 139
5 Maximally Distant Pairs of Trees ..... 141
5.1 Preliminaries ..... 141
5.2 Minor with respect to a pair of trees ..... 145
5.3 Principal sequence ..... 149
5.4 The principal minor ..... 155
5.5 Hybrid pre-rank and the principal minor ..... 159
5.6 Principal partition and Shannon's game ..... 164
5.7 Bibliographic notes ..... 167
Bibliography ..... 168
Index ..... 174

