Monique Florenzano · Cuong Le Van In cooperation with Pascal Gourdel

Finite Dimensional Convexity and Optimization

With 9 Figures



Springer

Contents

No	Notationx					
1.	Cor	$\mathbf{vexity in } \mathbb{R}^{n} \dots \dots$	1			
	1.1	Basic concepts	1			
		1.1.1 Line segment, convex set and convex hull	1			
		1.1.2 Line, affine set and affine hull	2			
		1.1.3 Affine maps	4			
		1.1.4 Affine independence, barycentric coordinates, dimension	5			
		1.1.5 Hyperplane	6			
		1.1.6 Convex cone and convex conic hull	7			
	1.2	Topological properties of convex sets	8			
		1.2.1 Closure and interior of a convex set	8			
		1.2.2 Relative interior and relative boundary of a convex set	11			
		1.2.3 Convex hull of a compact set	14			
	Exe	rcises	15			
2.	Sep	aration and Polarity	21			
	2.1	Separation of convex sets	21			
		2.1.1 Definitions	21			
		2.1.2 Projection on a closed convex set	22			
		2.1.3 Separation theorems	24			
	2.2	Polars of convex sets and orthogonal subspaces	28			
	2.3	Application to Minkowski-Farkas' lemma	29			
		2.3.1 Minkowski-Farkas' lemma	29			
		2.3.2 And its corollaries	32			
	Exe	rcises	33			
3.	\mathbf{Ext}	remal Structure of Convex Sets	37			
	3.1	Extreme points and faces of convex sets	37			
		3.1.1 Definitions	37			
		3.1.2 Krein-Milman's theorem	38			
	3.2	Application to linear inequalities. Weyl's theorem	40			
	3.3	Extreme points and extremal subsets of a polyhedral convex				
		set	45			

	Exe	rcises	47			
4.	Lin	ear Programming	51			
	4.1	Necessary and sufficient conditions of optimality	52			
		4.1.1 Necessary and sufficient conditions of optimality	52			
		4.1.2 Application to solving linear programming problems	53			
	4.2	The duality theorem of linear programming	54			
		4.2.1 Canonical form of the duality theorem	54			
		4.2.2 General form of the duality theorem	56			
		4.2.3 Application to nonhomogeneous Farkas' lemma	59			
	4.3	The simplex method	60			
		4.3.1 Extreme points of the set of nonnegative solutions to				
		a system of linear equations	60			
		4.3.2 Solving a linear programming problem in canonical form	61			
		4.3.3 Pivoting	62			
		4.3.4 Practicing the simplex method	67			
	\mathbf{Exe}	rcises	67			
-						
5.	5.1	Nvex Functions	73			
		Basic definitions and properties	73			
	$5.2 \\ 5.3$	Continuity theorems	76			
		Continuity properties of collections of convex functions	79 81			
	Exe	TCISES	01			
6.	Diff	ferential Theory of Convex Functions	87			
	6.1	The Hahn-Banach dominated extension theorem	88			
	6.2	Sublinear functions	89			
	6.3	Support functions	91			
	6.4	Directional derivatives	92			
	6.5	Subgradients and subdifferential of a convex function	93			
	6.6	Differentiability of convex functions	98			
	6.7	Differential continuity for convex functions 1	100			
	Exe	rcises 1	104			
7.	Cor	nvex Optimization With Convex Constraints	00			
1.	7.1	The minimum of a convex function $f : \mathbb{R}^n \to \mathbb{R}$				
	7.1	Kuhn-Tucker Conditions $\dots \dots \dots$				
	1.4	7.2.1 Necessary and sufficient condition for optimality				
		7.2.1 Recessary and sufficient condition for optimality \dots 17.2.2 Lagrangian of Problem (P) \dots 1				
		7.2.3 Duality in convex programming				
	7.3	Value function				
		rcises				
	Exe	נסספוט ו	20			

8.	Nor	1 Convex Optimization 1	129		
	8.1	Quasi-convex functions 1	129		
	8.2	Minimization of quasi-convex functions	134		
		Differentiable optimization 1			
		rcises 1			
А.	Appendix				
	A.1	Some preliminaries on topology	141		
	A.2	The Mean value theorem 1	142		
	A.3	The Local inversion theorem	142		
	A.4	The implicit functions theorem	145		
Bit	oliogi	caphy1	147		
Ind	lex	1	151		