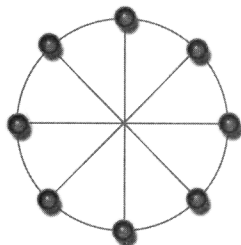


Introduction to



Phonons and Electrons

Liang-fu Lou

Former Member of Technical Staff
Northrop Grumman Corporation



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