

Handbook of Optical Systems

Edited by

Herbert Gross

Volume 2: Physical Image Formation

Wolfgang Singer, Michael Totzeck, Herbert Gross



WILEY-
VCH

WILEY-VCH Verlag GmbH & Co. KGaA

Contents

Introduction XIX

17	The Wave Equation	1
17.1	Introduction	2
17.2	From Maxwell to Helmholtz	2
17.2.1	Maxwell's Equations and the Inhomogeneous Wave Equation	2
17.2.2	Wave Equation in Homogeneous Media and the Scalar Wave Equation	4
17.2.3	The Dispersion Relation of the Harmonic Wave Solution	6
17.3	Elementary Waves in Free Space	9
17.3.1	The Electromagnetic Plane Wave	9
17.3.2	Spherical Wave	11
17.3.3	Dipole Wave	11
17.3.4	Radiated Field of a Harmonic Current Distribution	13
17.3.5	A Note on Plane and Spherical Waves	13
17.4	Energy, Irradiance and Intensity	14
17.5	The Angular Spectrum	17
17.5.1	Spatial Frequency Representation	17
17.5.2	Transformation of the Three-dimensional Spectrum into Two Dimensions	19
17.5.3	Free-space Propagation of Transverse Fields	20
17.5.4	Periodic Fields with Discrete Spectra	22
17.5.5	Boundary Conditions and the Spatial Frequency Spectrum	23
17.5.6	Vector Field Representation by Spatial Frequencies	24
17.6	Evanescent Waves	26
17.7	Approximative Solutions to the Wave Equation	28
17.7.1	Geometrical Optics and the Eikonal Equation	28
17.7.2	Paraxial Wave Equation	29
17.7.3	Transport of Intensity	30
17.7.4	Gaussian Beams	31
17.7.5	Ray Equivalent of Gaussian Beams	36
17.7.6	Gaussian Beams in Two Dimensions	37
17.8	Literature	39

18	Scalar Diffraction	41
18.1	Introduction	42
18.2	Kirchhoff Diffraction Integral	44
18.2.1	Inconsistency of the Kirchhoff Diffraction Integral	48
18.3	1 st and 2 nd Rayleigh–Sommerfeld Diffraction Integral	48
18.4	Two-dimensional Diffraction	50
18.5	Huygens Principle	52
18.6	Fourier Space Formulation	54
18.7	Examples of Scalar Diffraction Patterns	57
18.7.1	Diffraction Fields Behind Slits	57
18.7.2	Diffraction by a Rectangular Aperture	59
18.8	Fresnel Diffraction	60
18.8.1	Computation	61
18.8.2	Validity	62
18.9	Collin’s Fresnel Diffraction Integral	64
18.9.1	Definition	64
18.9.2	Example	67
18.10	Fraunhofer Diffraction	69
18.11	Grating Diffraction	71
18.11.1	Ronchi Grating	71
18.11.2	The Sinusoidal Phase Grating and Surface Fabrication Errors	76
18.12	Scalar Diffraction at Dielectric Objects	79
18.13	Babinet’s Principle	82
18.14	Scalar Scattering	85
18.15	Boundary Diffraction Waves	89
18.15.1	Geometrical Theory of Diffraction	90
18.15.2	An Empirical Boundary Diffraction Wave	94
18.16	Literature	96
19	Interference and Coherence	99
19.1	Basic Principles	100
19.1.1	Introduction	100
19.1.2	Two-beam Interference and Double Slit Diffraction	102
19.1.3	Contributions of Different Points of the Light Source	105
19.1.4	The High-frequency Term	107
19.1.5	The Low-frequency Term	108
19.1.6	Different Light Source Points with Statistical Phase	109
19.2	Mathematical Description of Coherence	113
19.2.1	Coherence Function	113
19.2.2	Wigner Distribution Function	116
19.2.3	Moments of the Wigner Distribution Function	120
19.2.4	Smoothing of the Wigner Distribution Function and Diffraction Focus	121
19.2.5	Wigner Distribution Function of Coherent Fields	122
19.2.6	Ambiguity Function	123

19.2.7	The Characterizing Functions in their Context	125
19.3	Temporal Coherence	126
19.3.1	Superposition of Signals with Different Frequency	126
19.3.2	Spectral Distribution of a Light Source	127
19.3.3	Bandwidth-limited Signals	128
19.3.4	Axial Coherence Length	130
19.3.5	Thermal Light Sources	133
19.3.6	Temporal Coherence in the Michelson Interferometer	134
19.4	Spatial Coherence	135
19.4.1	Introduction	135
19.4.2	Propagation of the Coherence Function	138
19.4.3	Van Cittert-Zernike Theorem	140
19.4.4	The Coherence Function of a Circular Source	140
19.4.5	Coherence Function behind a Double Slit	143
19.4.6	Propagation of the Wigner Distribution Function	146
19.5	Gaussian Schell Beams	149
19.5.1	Definition of Gaussian Schell Beams	149
19.5.2	Coherence and Wigner Functions of Gaussian Schell Beams	154
19.5.3	Basis Mode Expansion of Partial Coherent Fields	156
19.6	Statistical Optics and Speckle	159
19.6.1	Photon Statistics	159
19.6.2	The Speckle Effect	161
19.6.3	Speckle Parameters and Surface Structure	163
19.6.4	Computation of Speckle Effects	165
19.6.5	Speckle Reduction	169
19.7	Array Homogenizer	172
19.7.1	Setup of the System	172
19.7.2	Pupil Filling	175
19.7.3	Coherence Effects	176
19.7.4	Example Calculation	177
19.8	Miscellaneous	179
19.8.1	General Coherence Length	179
19.8.2	General Degree of Coherence	182
19.8.3	Coherence and Polarization	183
19.9	Literature	184
20	The Geometrical Optical Description and Incoherent Imaging	187
20.1	Introduction	188
20.2	Characteristic Functions	189
20.2.1	Geometrical Optics and the Wave Equation	189
20.2.2	The Characteristic Functions	191
20.2.3	Geometrical-optical imaging	194
20.2.4	The Canonical Pupil	196
20.2.5	A Note on Diffractive Optical Elements	199

20.3	The Ideal Wave-optical Image of a Point and Geometrical-optical Image Formation	200
20.3.1	The Scalar Luneburg Integral	200
20.3.2	Energy Discussions for Optical Imaging	204
20.3.3	The Airy Disc	206
20.3.4	Incoherent Resolution	210
20.4	Aberrations of Optical Systems	211
20.4.1	The Small-aberration Limit: The Strehl Ratio	211
20.4.2	Expansion of the Wave-front Error into Zernike Polynomials	212
20.4.3	Point Images for Different Aberrations	217
20.4.4	Distortion, Defocus and Astigmatism	219
20.4.5	Spherical Aberrations Z_0 , Coma Z_7 and Z_8	220
20.4.6	Line of Sight	221
20.4.7	Wave Aberrations for Annular Pupils	224
20.4.8	Extended Zernike Expansion	227
20.5	Helmholtz–Lagrange Invariant and Phase-space Description	231
20.5.1	The Phase Space	231
20.5.2	The Resolution Limit in the Space Domain and in the Spatial Frequency Domain	234
20.5.3	The Space–Bandwidth Product	236
20.6	Literature	237
21	The Abbe Theory of Imaging	239
21.1	Introduction	240
21.2	Phenomenological Description of Imaging	244
21.2.1	The Explanation of Image Formation According to Abbe and the Abbe Resolution	244
21.2.2	The Information About an Object Contained in an Image	249
21.2.3	Koehler Illumination and the Visibility	252
21.2.4	The Siedentopf Illumination Principle	255
21.2.5	Imaging with Different Colours	259
21.2.6	Aplanatic Correction and Geometrical Optics	260
21.3	The Mathematical Description of Fourier Optical Imaging	262
21.3.1	Imaging with Uncorrelated Light Sources	262
21.3.2	Consideration of Magnification	267
21.4	Coherence in Imaging	269
21.4.1	The Coherent Image	269
21.4.2	Incoherent Imaging	272
21.4.3	One-Dimensional Incoherent Imaging	273
21.4.4	Systems with Rotational Symmetry	275
21.4.5	Conditions for Incoherent, Partially Coherent and Coherent Imaging	277
21.4.6	Imaging with Correlated Light Sources	280
21.5	Literature	281

22	Coherence Theory of Optical Imaging	283
22.1	Introduction	284
22.2	Theoretical Description of Partially Coherent Image Formation	284
22.2.1	Hopkins Transmission Cross Coefficient	284
22.2.2	Image Fidelity	287
22.2.3	Hopkins Formalism for Periodic Objects	288
22.2.4	Aberrations in the Linear Grating Image	293
22.3	The Coherence Function and the Coherence Transfer Function	296
22.4	The Phase Space Description	300
22.4.1	Transformation of Coherence and Wigner Distribution Function	300
22.4.2	Propagation of the Wigner Distribution Function in Free Space	303
22.4.3	Compilation of the Transformations	307
22.5	Optical Imaging in the Presence of Aberrations	309
22.5.1	Linear Systems and Classification of Aberrations	309
22.5.2	Random Non-stationary Aberrations: Stray Light and Flare	314
22.6	Literature	317
23	Three-dimensional Imaging	319
23.1	Introduction	320
23.2	The Ewald Sphere and the Generalized Pupil	321
23.2.1	The Ewald Sphere	321
23.2.2	The Generalized Aperture and the Three-dimensional Point-spread Function	322
23.3	The Three-dimensional Transfer Function	327
23.3.1	Born Approximation and the Laue Equation	327
23.3.2	Dändliker's Representation and the Shape of the Three-dimensional Transfer Function	330
23.3.3	Resolution, Depth Resolution and Depth of Focus	335
23.3.4	3D-Transfer Functions in Microscopy	338
23.3.5	Magnification and a Comment on Absolute Instruments	340
23.4	Selected Examples of the Three-Dimensional Transfer Function	343
23.4.1	Transfer Function for Incoherent Imaging with $\sigma=1$	343
23.4.2	Partial Coherent Image Examples	344
23.4.3	'Tayloring' of the 3D-Transfer Function	346
23.4.5	Influence of Aberrations	351
23.5	Literature	352
24	Image Examples of Selected Objects	355
24.1	Introduction	356
24.2	Two-point Resolution	356
24.2.1	Incoherent Versus Coherent Two-point Resolution	356
24.2.2	Image of a Double Slit for Coherent and Incoherent Illumination	360
24.2.3	Phase Shift and Oblique Illumination	364
24.3	The Image of an Edge	365
24.3.1	The Coherent Image of an Amplitude and Phase Edge	365

24.3.2	The Incoherent Image of an Amplitude Edge	369
24.3.3	Partially Coherent Edge Image	370
24.3.4	The Determination of the Optical Transfer Function from the Edge Image	375
24.4	The Line Image	376
24.4.1	The Line Image of a Rotational-symmetrical Lens	376
24.4.2	Coherent Line or Slit Image	377
24.4.3	Incoherent Line or Slit Image	380
24.5	The Grating Image	381
24.5.1	The Coherent Linear Grating Image	381
24.5.2	The Coherent Grating Image with Aberrations	384
24.5.3	The Influence of the Coherence Parameter σ on the Grating Image	386
24.5.4	Influence of the Shape of the Effective Light Source on the Grating Image	389
24.5.5	Wigner Distribution Function for Gratings, Talbot Effect and Propagation-invariant Fields	394
24.6	Pinhole Imaging and Quasi-point Sources	399
24.6.1	Introduction	399
24.6.2	Incoherent Image of a Circular Object	400
24.6.3	Quasi-point Source	402
24.6.4	Pinhole with Coherent Illumination	404
24.6.5	Pinhole with Partial Coherent Illumination	405
24.6.6	Defocusing Planes and Deconvolution	406
24.7	Literature	407
25	Special System Examples and Applications	409
25.1	Introduction	410
25.2	Point-spread Functions for Annular Pupils	410
25.2.1	Introduction	410
25.2.2	Annular Pupils, Central Obscuration and Pupil Filters	411
25.3	Point-spread Functions of Non-uniform Illuminated Pupils	416
25.3.1	Introduction	416
25.3.2	General Gaussian Apodization	417
25.3.3	Gaussian Profile with Truncation	418
25.4	Engineering of the Point-spread Function by Pupil Masks	423
25.4.1	Introduction	423
25.4.2	Characterization of the Three-dimensional Point-spread Function	423
25.4.3	Characterization of Extended Depth of Focus	426
25.4.4	Relation Between Axial and Transverse Resolution	427
25.4.5	Ambiguity Function as Defocussed Transfer Function	429
25.4.6	Image Multiplexing	430
25.4.7	Fundamental Relationships	432
25.4.8	Calculation of Masks	432
25.5	Special Pupil Masks	433
25.5.1	Introduction	433

25.5.2	Phase Masks According to Toraldo	434
25.5.3	Logarithmic Phase Mask	435
25.5.4	Chirped Ring Pupil	437
25.5.5	Complex Filter Described by Zernike Expansions	439
25.5.6	Cubic Phase Plates for Extended Depth of Focus	442
25.5.7	Structured Illumination	447
25.6	Selected Practical Applications for Pupil Filtering Techniques	450
25.6.1	Phase Contrast Filtering, Dark-field Illumination	450
25.6.2	Frequency Doubling	453
25.6.3	Defect Filtering	455
25.6.4	Ronchi Test	456
25.7	Literature	463
26	Polarization	465
26.1	Introduction	467
26.2	Polarization States	467
26.2.1	Representation of Polarization States	468
26.2.2	Jones Vector	468
26.2.3	Ellipse of Polarization	470
26.2.4	Orthogonal Jones Vectors	471
26.2.5	Jones Vectors in Different Bases	472
26.2.6	Unpolarized Light	472
26.2.7	Partial Polarization	473
26.2.8	Polarization Matrix	473
26.2.9	Stokes Vector	475
26.2.10	Poincaré Sphere	478
26.3	Jones Matrix	479
26.3.1	Definition	479
26.3.2	Jones Matrix Acting on a Jones Vector	480
26.3.3	Succession of Jones Matrices	480
26.3.4	Jones Matrix Acting on a Polarization Matrix	481
26.3.5	Examples of Jones Matrices	481
26.3.6	Rotated and Mirrored Jones Matrix	482
26.3.7	Jones Matrix for Different Basis Polarization States	483
26.3.8	Eigenpolarizations of a Jones Matrix	483
26.3.9	Jones Matrix of a Retarder	484
26.3.10	Jones Matrix of a Partial Polarizer	487
26.3.11	Pauli's Spin Matrices	489
26.3.12	Jones Matrix Decomposition	489
26.4	Müller Matrix	491
26.4.1	Definition	491
26.4.2	Examples	492
26.5	Müller–Jones Matrix	493
26.6	Light in Anisotropic Media	494
26.6.1	Anisotropic Media	494

26.6.2	Principal Refractive Indices of an Anisotropic Medium Without Spatial Dispersion and Optical Activity	495
26.6.3	Fresnel Ellipsoid	496
26.6.4	Index Ellipsoid	497
26.6.5	Types of Birefringent Media	497
26.7	Eigenwaves in Anisotropic Media	501
26.7.1	Plane Waves in Anisotropic Media	501
26.7.2	Eigenwaves and their Polarization	502
26.7.3	Properties of the Eigenpolarizations	506
26.7.4	The Intersection Ellipse	506
26.8	Jones Matrix of Propagation	507
26.9	Jones Matrices of Propagation for Common Media	508
26.9.1	Eigenpolarizations and ν -values	508
26.9.2	Coordinate Systems	509
26.9.3	Uniaxial Crystal	509
26.9.4	Biaxial Crystal	510
26.9.5	CaF ₂ with Spatial Dispersion at $\lambda = 193$ nm	511
26.10	Beam-splitting in an Anisotropic Medium	511
26.11	Examples of Polarization-optical Elements	516
26.11.1	Quarter-wave and Half-wave Retarder	516
26.11.2	Babinet–Soleil Compensator	516
26.11.3	Faraday Rotator	518
26.11.4	Brewster Plate	519
26.12	Literature	520
27	Vector Diffraction	523
27.1	Introduction	524
27.2	Focus Computation for Polarized Fields	525
27.2.1	Geometry for Focus Computation	525
27.2.2	Richards–Wolf integral	526
27.2.3	Plane Wave Expansion	531
27.2.4	Focus Fields for Various Input Polarizations	533
27.3	Vector Kirchhoff Diffraction Integral	538
27.4	Analytical Solutions	538
27.4.1	Plane Interface: Fresnel's Equations	540
27.4.2	Diffraction at a Circular Cylinder	542
27.4.3	Mie Scattering	547
27.5	Numerical Methods for Arbitrary Structures	553
27.6	Coupled Dipole Method	553
27.7	Integral Equation Approach and Moment Method	555
27.7.1	The Moment Method	555
27.7.2	Form of Scattering Operator	556
27.7.3	Scattering in Three-layer Medium	557
27.8	Fourier Modal Method	563
27.8.1	Theory	563

27.8.2	Diffraction Efficiency	568
27.9	Finite-difference Method	568
27.9.1	Boundary Conditions	570
27.9.2	Implicit Paraxial Wave Equation in Two Dimensions	572
27.9.3	Paraxial Wave Equation in Cylindrical Coordinates	572
27.9.4	ADI-formulation of the Paraxial Wave Equation in Three Dimensions	575
27.9.5	Split-step-beam Propagation Method	576
27.10	Rigorous Diffraction in Optical Imaging	579
27.10.1	Dielectrics and Metals	579
27.11	Simulation of Polarized Imaging by use of Rigorous Diffraction	583
27.12	Literature	587
28	Polarization and Optical Imaging	589
28.1	Introduction	590
28.2	The Image-forming Field	590
28.3	Interference of Electromagnetic Waves	592
28.3.1	Two-beam Vector Interference	592
28.3.2	Contrast for High-NA, s- and p-polarization	593
28.3.3	Influence of Recording Medium	594
28.3.4	Vector Effect in Optical Microscopy	595
28.3.5	Vector Effect in Optical Lithography	595
28.4	Polarized Ray Trace	596
28.4.1	Definition of Ray, Beam and Path	597
28.4.2	Ray-splitting at Anisotropic Elements	597
28.4.3	Refraction and Reflection at Birefringent Interfaces	598
28.4.4	The Single-path Approximation	599
28.5	Optical Systems with Polarization Effects	604
28.6	Polarized Imaging Model	605
28.6.1	Scalar Image	606
28.6.2	Vector Image for Completely Polarized Illumination	607
28.6.3	Vector Image for Partially Polarized Illumination	609
28.7	Vector Point-spread Function	610
28.7.1	VPSF for Complete Polarization	610
28.7.2	VPSF for Unpolarized Illumination	611
28.8	Polarized Optical Transfer Function	612
28.8.1	Polarized Illumination	612
28.8.2	Unpolarized Illumination	612
28.9	Jones Matrix Pupil	612
28.9.1	Definition for Completely Polarized Illumination	613
28.9.2	Separation of a Scalar Factor	614
28.9.3	Decomposition into Retardance and Diattenuation	615
28.9.4	Example	616
28.10	Jones Matrix Pupils in the Polarization Matrix Calculus	617
28.11	Jones-matrix-based System Optimization	619

28.12	Aberrations of the Transmitted Wavefront	620
28.13	Jones–Zernike Wavefront Aberrations	621
28.13.1	Principle of the Modal Characterization of a Jones Pupil	621
28.13.2	Jones–Zernike Expansion	621
28.13.3	Properties of the Jones–Zernike Polynomials	623
28.14	Literature	625
A1	Mathematical Appendix	627
A.1	Linear Systems	629
A.2	Fourier Series and Fourier Integral	631
A.2.1	Compilation of Basic Properties of the Fourier Transform	632
A.2.2	Special Functions and their Fourier Transforms	634
A.3	Convolution and Correlation	637
A.3.1	Convolution	637
A.3.2	Correlation	637
A.3.3	Power Spectral Density and RMS Value	638
A.4	Discrete Signals	639
A.4.1	The Sampling Theorem	639
A.4.2	Leakage	641
A.4.3	Indexing of the Numerical Discrete Fast Fourier Transform	642
A.5	z-Transform	644
A.5.1	Definition	644
A.5.2	Numerical Evaluation of the z-transform	646
A.5.3	Sinc Interpolation	648
A.6	Hankel Transform	648
A.6.1	Definition	648
A.6.2	Numerical Computation	649
A.7	Practical Calculation of Diffraction Integrals	655
A.7.1	The Oscillation Problem	655
A.7.2	Spatial and Spectral Resolution	660
A.7.3	Periodic Boundary Conditions	662
A.7.4	x-z Sampling of the Ewald Sphere	663
A.7.5	Equivalent Diffraction Setups	663
A.7.6	Optimal Conditioning of the Fresnel Diffraction	666
A.7.7	Numerical Algorithms	669
A.7.8	Fresnel Integrals	672
A.8	Orthogonal Polynomials on Rectangular Domains	675
A.8.1	Chebyshev Polynomials	675
A.8.2	One-dimensional Legendre Polynomials	677
A.8.3	Two-dimensional Chebyshev Polynomials	678
A.8.4	Legendre Polynomials in Two Dimensions	679
A.9	Literature	683
Index		685