

# Handbook of Optical Systems

*Edited by*  
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Volume 3: Aberration Theory and Correction  
of Optical Systems  
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