

World Scientific Lecture Notes in Physics Vol. 37

**Distributions, Fourier Transforms
and
Some of Their Applications to Physics**

Thomas Schücker

Institute for Theoretical Physics
University of Heidelberg



World Scientific

Singapore • New Jersey • London • Hong Kong

Contents

Preface	vii
1 Functions of a complex variable	1
1.1 Complex numbers	1
1.2 Roots	4
1.3 Complex functions	6
1.4 Differentiation	7
1.5 Integration	10
1.6 Residues	15
1.7 Applications to the real world	18
2 Distributions	24
2.1 Motivation	24
2.2 Weak convergence	27
2.3 Operations with distributions	32
2.4 Correspondence between functions and distributions	36
2.5 Sequences of distributions	40
2.6 Green functions	42
2.7 Distributions in higher dimensions	49
3 Fourier series	55
3.1 Periodic functions	55
3.2 Sequences of functions, different notions of convergence	56
3.3 Real and complex harmonics	59
3.4 A scalar product and the Fourier coefficients	60
3.5 Pointwise convergence of the Fourier series	64
3.6 Fourier series of periodic distributions	65
3.7 Forced oscillations	70
3.8 The string	71
4 Fourier transforms	76
4.1 Heuristics	76
4.2 The Fourier transform of functions	77
4.3 The Fourier transform of distributions	84
4.4 Calculating a Green function	89
4.5 Fourier transforms in higher dimensions	91
4.6 The Laplace transform	94

5	Some linear differential operators and their Green functions	96
5.1	The Laplace equation	97
5.2	The heat equation	101
5.3	The wave equation	104
5.4	The Klein-Gordon equation	110
5.5	The Dirac equation	113
6	Linear algebra in infinite dimensions	116
6.1	Complex vector spaces	116
6.2	Operators	118
6.3	Scalar products	121
6.4	Orthonormal bases	126
6.5	Hilbert spaces	132
6.6	Self-adjoint operators	134
7	Systems of special functions	140
7.1	Hermite functions and polynomials	140
7.2	Other orthogonal polynomials	142
7.3	The spherical harmonics	144
7.4	Stokes' law of hydrodynamics	146
7.5	Bessel functions	152
7.6	The circular membrane	154
	References	157
	Bibliography	159
	Notation	161
	Index	165