

Mathematical Physics of Quantum Wires and Devices

From Spectral Resonances to Anderson Localization

by

Norman E. Hurt

*Zeta Associates, Inc.,
Fairfax, VA, U.S.A.*



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