

Numerical Methods for Engineers and Scientists

***Second Edition
Revised and Expanded***

Joe D. Hoffman

*Department of Mechanical Engineering
Purdue University
West Lafayette, Indiana*



MARCEL DEKKER, INC.

NEW YORK • BASEL

Contents

| | |
|---|-----------|
| Preface | v |
| Chapter 0. Introduction | 1 |
| 0.1. Objectives and Approach | 1 |
| 0.2. Organization of the Book | 2 |
| 0.3. Examples | 2 |
| 0.4. Programs | 3 |
| 0.5. Problems | 3 |
| 0.6. Significant Digits, Precision, Accuracy, Errors, and Number Representation | 4 |
| 0.7. Software Packages and Libraries | 6 |
| 0.8. The Taylor Series and the Taylor Polynomial | 7 |
| Part I. Basic Tools of Numerical Analysis | 11 |
| I.1. Systems of Linear Algebraic Equations | 11 |
| I.2. Eigenproblems | 13 |
| I.3. Roots of Nonlinear Equations | 14 |
| I.4. Polynomial Approximation and Interpolation | 14 |
| I.5. Numerical Differentiation and Difference Formulas | 15 |
| I.6. Numerical Integration | 16 |
| I.7. Summary | 16 |
| Chapter 1. Systems of Linear Algebraic Equations | 17 |
| 1.1. Introduction | 18 |
| 1.2. Properties of Matrices and Determinants | 21 |
| 1.3. Direct Elimination Methods | 30 |
| 1.4. LU Factorization | 45 |
| 1.5. Tridiagonal Systems of Equations | 49 |
| 1.6. Pitfalls of Elimination Methods | 52 |
| 1.7. Iterative Methods | 59 |
| 1.8. Programs | 67 |
| 1.9. Summary | 76 |
| Exercise Problems | 77 |
| Chapter 2. Eigenproblems | 81 |
| 2.1. Introduction | 81 |
| 2.2. Mathematical Characteristics of Eigenproblems | 85 |
| 2.3. The Power Method | 89 |

| | | |
|--|--|-----|
| 2.4. | The Direct Method | 101 |
| 2.5. | The QR Method | 104 |
| 2.6. | Eigenvectors | 110 |
| 2.7. | Other Methods | 111 |
| 2.8. | Programs | 112 |
| 2.9. | Summary | 118 |
| | Exercise Problems | 119 |
| Chapter 3. Nonlinear Equations | | 127 |
| 3.1. | Introduction | 127 |
| 3.2. | General Features of Root Finding | 130 |
| 3.3. | Closed Domain (Bracketing) Methods | 135 |
| 3.4. | Open Domain Methods | 140 |
| 3.5. | Polynomials | 155 |
| 3.6. | Pitfalls of Root Finding Methods and Other Methods of Root Finding | 167 |
| 3.7. | Systems of Nonlinear Equations | 169 |
| 3.8. | Programs | 173 |
| 3.9. | Summary | 179 |
| | Exercise Problems | 181 |
| Chapter 4. Polynomial Approximation and Interpolation | | 187 |
| 4.1. | Introduction | 188 |
| 4.2. | Properties of Polynomials | 190 |
| 4.3. | Direct Fit Polynomials | 197 |
| 4.4. | Lagrange Polynomials | 198 |
| 4.5. | Divided Difference Tables and Divided Difference Polynomials | 204 |
| 4.6. | Difference Tables and Difference Polynomials | 208 |
| 4.7. | Inverse Interpolation | 217 |
| 4.8. | Multivariate Approximation | 218 |
| 4.9. | Cubic Splines | 221 |
| 4.10. | Least Squares Approximation | 225 |
| 4.11. | Programs | 235 |
| 4.12. | Summary | 242 |
| | Exercise Problems | 243 |
| Chapter 5. Numerical Differentiation and Difference Formulas | | 251 |
| 5.1. | Introduction | 251 |
| 5.2. | Unequally Spaced Data | 254 |
| 5.3. | Equally Spaced Data | 257 |
| 5.4. | Taylor Series Approach | 264 |
| 5.5. | Difference Formulas | 270 |
| 5.6. | Error Estimation and Extrapolation | 270 |
| 5.7. | Programs | 273 |
| 5.8. | Summary | 279 |
| | Exercise Problems | 279 |
| Chapter 6. Numerical Integration | | 285 |
| 6.1. | Introduction | 285 |
| 6.2. | Direct Fit Polynomials | 288 |
| 6.3. | Newton-Cotes Formulas | 290 |

| | | |
|---|--|------------|
| 6.4. | Extrapolation and Romberg Integration | 297 |
| 6.5. | Adaptive Integration | 299 |
| 6.6. | Gaussian Quadrature | 302 |
| 6.7. | Multiple Integrals | 306 |
| 6.8. | Programs | 311 |
| 6.9. | Summary | 315 |
| | Exercise Problems | 316 |
| Part II. Ordinary Differential Equations | | 323 |
| II.1. | Introduction | 323 |
| II.2. | General Features of Ordinary Differential Equations | 323 |
| II.3. | Classification of Ordinary Differential Equations | 325 |
| II.4. | Classification of Physical Problems | 326 |
| II.5. | Initial-Value Ordinary Differential Equations | 327 |
| II.6. | Boundary-Value Ordinary Differential Equations | 330 |
| II.7. | Summary | 332 |
| Chapter 7. One-Dimensional Initial-Value Ordinary Differential Equations | | 335 |
| 7.1. | Introduction | 336 |
| 7.2. | General Features of Initial-Value ODEs | 340 |
| 7.3. | The Taylor Series Method | 343 |
| 7.4. | The Finite Difference Method | 346 |
| 7.5. | The First-Order Euler Methods | 352 |
| 7.6. | Consistency, Order, Stability, and Convergence | 359 |
| 7.7. | Single-Point Methods | 364 |
| 7.8. | Extrapolation Methods | 378 |
| 7.9. | Multipoint Methods | 381 |
| 7.10. | Summary of Methods and Results | 391 |
| 7.11. | Nonlinear Implicit Finite Difference Equations | 393 |
| 7.12. | Higher-Order Ordinary Differential Equations | 397 |
| 7.13. | Systems of First-Order Ordinary Differential Equations | 398 |
| 7.14. | Stiff Ordinary Differential Equations | 401 |
| 7.15. | Programs | 408 |
| 7.16. | Summary | 414 |
| | Exercise Problems | 416 |
| Chapter 8. One-Dimensional Boundary-Value Ordinary Differential Equations | | 435 |
| 8.1. | Introduction | 436 |
| 8.2. | General Features of Boundary-Value ODEs | 439 |
| 8.3. | The Shooting (Initial-Value) Method | 441 |
| 8.4. | The Equilibrium (Boundary-Value) Method | 450 |
| 8.5. | Derivative (and Other) Boundary Conditions | 458 |
| 8.6. | Higher-Order Equilibrium Methods | 466 |
| 8.7. | The Equilibrium Method for Nonlinear Boundary-Value Problems | 471 |
| 8.8. | The Equilibrium Method on Nonuniform Grids | 477 |
| 8.9. | Eigenproblems | 480 |
| 8.10. | Programs | 483 |
| 8.11. | Summary | 488 |
| | Exercise Problems | 490 |

| | |
|---|------------|
| Part III. Partial Differential Equations | 501 |
| III.1. Introduction | 501 |
| III.2. General Features of Partial Differential Equations | 502 |
| III.3. Classification of Partial Differential Equations | 504 |
| III.4. Classification of Physical Problems | 511 |
| III.5. Elliptic Partial Differential Equations | 516 |
| III.6. Parabolic Partial Differential Equations | 519 |
| III.7. Hyperbolic Partial Differential Equations | 520 |
| III.8. The Convection-Diffusion Equation | 523 |
| III.9. Initial Values and Boundary Conditions | 524 |
| III.10. Well-Posed Problems | 525 |
| III.11. Summary | 526 |
| Chapter 9. Elliptic Partial Differential Equations | 527 |
| 9.1. Introduction | 527 |
| 9.2. General Features of Elliptic PDEs | 531 |
| 9.3. The Finite Difference Method | 532 |
| 9.4. Finite Difference Solution of the Laplace Equation | 536 |
| 9.5. Consistency, Order, and Convergence | 543 |
| 9.6. Iterative Methods of Solution | 546 |
| 9.7. Derivative Boundary Conditions | 550 |
| 9.8. Finite Difference Solution of the Poisson Equation | 552 |
| 9.9. Higher-Order Methods | 557 |
| 9.10. Nonrectangular Domains | 562 |
| 9.11. Nonlinear Equations and Three-Dimensional Problems | 570 |
| 9.12. The Control Volume Method | 571 |
| 9.13. Programs | 575 |
| 9.14. Summary | 580 |
| Exercise Problems | 582 |
| Chapter 10. Parabolic Partial Differential Equations | 587 |
| 10.1. Introduction | 587 |
| 10.2. General Features of Parabolic PDEs | 591 |
| 10.3. The Finite Difference Method | 593 |
| 10.4. The Forward-Time Centered-Space (FTCS) Method | 599 |
| 10.5. Consistency, Order, Stability, and Convergence | 605 |
| 10.6. The Richardson and DuFort-Frankel Methods | 611 |
| 10.7. Implicit Methods | 613 |
| 10.8. Derivative Boundary Conditions | 623 |
| 10.9. Nonlinear Equations and Multidimensional Problems | 625 |
| 10.10. The Convection-Diffusion Equation | 629 |
| 10.11. Asymptotic Steady State Solution to Propagation Problems | 637 |
| 10.12. Programs | 639 |
| 10.13. Summary | 645 |
| Exercise Problems | 646 |
| Chapter 11. Hyperbolic Partial Differential Equations | 651 |
| 11.1. Introduction | 651 |
| 11.2. General Features of Hyperbolic PDEs | 655 |

| | |
|--|-----|
| 11.3. The Finite Difference Method | 657 |
| 11.4. The Forward-Time Centered-Space (FTCS) Method and the Lax Method | 659 |
| 11.5. Lax-Wendroff Type Methods | 655 |
| 11.6. Upwind Methods | 673 |
| 11.7. The Backward-Time Centered-Space (BTCS) Method | 677 |
| 11.8. Nonlinear Equations and Multidimensional Problems | 682 |
| 11.9. The Wave Equation | 683 |
| 11.10. Programs | 691 |
| 11.11. Summary | 701 |
| Exercise Problems | 702 |
| Chapter 12. The Finite Element Method | 711 |
| 12.1. Introduction | 711 |
| 12.2. The Rayleigh-Ritz, Collocation, and Galerkin Methods | 713 |
| 12.3. The Finite Element Method for Boundary Value Problems | 724 |
| 12.4. The Finite Element Method for the Laplace (Poisson) Equation | 739 |
| 12.5. The Finite Element Method for the Diffusion Equation | 752 |
| 12.6. Programs | 759 |
| 12.7. Summary | 769 |
| Exercise Problems | 770 |
| References | 775 |
| Answers to Selected Problems | 779 |
| Index | 795 |