

Optimal State Estimation

Kalman, H_∞ , and Nonlinear Approaches

Dan Simon

Cleveland State University



A JOHN WILEY & SONS, INC., PUBLICATION

CONTENTS

Acknowledgments	xiii
Acronyms	xv
List of algorithms	xvii
Introduction	xxi

PART I INTRODUCTORY MATERIAL

1 Linear systems theory	3
1.1 Matrix algebra and matrix calculus	4
1.1.1 Matrix algebra	6
1.1.2 The matrix inversion lemma	11
1.1.3 Matrix calculus	14
1.1.4 The history of matrices	17
1.2 Linear systems	18
1.3 Nonlinear systems	22
1.4 Discretization	26
1.5 Simulation	27
1.5.1 Rectangular integration	29
1.5.2 Trapezoidal integration	29
1.5.3 Runge–Kutta integration	31
1.6 Stability	33

1.6.1	Continuous-time systems	33
1.6.2	Discrete-time systems	37
1.7	Controllability and observability	38
1.7.1	Controllability	38
1.7.2	Observability	40
1.7.3	Stabilizability and detectability	43
1.8	Summary	45
	Problems	45
2	Probability theory	49
2.1	Probability	50
2.2	Random variables	53
2.3	Transformations of random variables	59
2.4	Multiple random variables	61
2.4.1	Statistical independence	62
2.4.2	Multivariate statistics	65
2.5	Stochastic Processes	68
2.6	White noise and colored noise	71
2.7	Simulating correlated noise	73
2.8	Summary	74
	Problems	75
3	Least squares estimation	79
3.1	Estimation of a constant	80
3.2	Weighted least squares estimation	82
3.3	Recursive least squares estimation	84
3.3.1	Alternate estimator forms	86
3.3.2	Curve fitting	92
3.4	Wiener filtering	94
3.4.1	Parametric filter optimization	96
3.4.2	General filter optimization	97
3.4.3	Noncausal filter optimization	98
3.4.4	Causal filter optimization	100
3.4.5	Comparison	101
3.5	Summary	102
	Problems	102
4	Propagation of states and covariances	107
4.1	Discrete-time systems	107
4.2	Sampled-data systems	111
4.3	Continuous-time systems	114

4.4	Summary	117
	Problems	117

PART II THE KALMAN FILTER

5	The discrete-time Kalman filter	123
5.1	Derivation of the discrete-time Kalman filter	124
5.2	Kalman filter properties	129
5.3	One-step Kalman filter equations	131
5.4	Alternate propagation of covariance	135
	5.4.1 Multiple state systems	135
	5.4.2 Scalar systems	137
5.5	Divergence issues	139
5.6	Summary	144
	Problems	145
6	Alternate Kalman filter formulations	149
6.1	Sequential Kalman filtering	150
6.2	Information filtering	156
6.3	Square root filtering	158
	6.3.1 Condition number	159
	6.3.2 The square root time-update equation	162
	6.3.3 Potter's square root measurement-update equation	165
	6.3.4 Square root measurement update via triangularization	169
	6.3.5 Algorithms for orthogonal transformations	171
6.4	U-D filtering	174
	6.4.1 U-D filtering: The measurement-update equation	174
	6.4.2 U-D filtering: The time-update equation	176
6.5	Summary	178
	Problems	179
7	Kalman filter generalizations	183
7.1	Correlated process and measurement noise	184
7.2	Colored process and measurement noise	188
	7.2.1 Colored process noise	188
	7.2.2 Colored measurement noise: State augmentation	189
	7.2.3 Colored measurement noise: Measurement differencing	190
7.3	Steady-state filtering	193
	7.3.1 α - β filtering	199
	7.3.2 α - β - γ filtering	202
	7.3.3 A Hamiltonian approach to steady-state filtering	203
7.4	Kalman filtering with fading memory	208

7.5	Constrained Kalman filtering	212
7.5.1	Model reduction	212
7.5.2	Perfect measurements	213
7.5.3	Projection approaches	214
7.5.4	A pdf truncation approach	218
7.6	Summary	223
	Problems	225
8	The continuous-time Kalman filter	229
8.1	Discrete-time and continuous-time white noise	230
8.1.1	Process noise	230
8.1.2	Measurement noise	232
8.1.3	Discretized simulation of noisy continuous-time systems	232
8.2	Derivation of the continuous-time Kalman filter	233
8.3	Alternate solutions to the Riccati equation	238
8.3.1	The transition matrix approach	238
8.3.2	The Chandrasekhar algorithm	242
8.3.3	The square root filter	246
8.4	Generalizations of the continuous-time filter	247
8.4.1	Correlated process and measurement noise	248
8.4.2	Colored measurement noise	249
8.5	The steady-state continuous-time Kalman filter	252
8.5.1	The algebraic Riccati equation	253
8.5.2	The Wiener filter is a Kalman filter	257
8.5.3	Duality	258
8.6	Summary	259
	Problems	260
9	Optimal smoothing	263
9.1	An alternate form for the Kalman filter	265
9.2	Fixed-point smoothing	267
9.2.1	Estimation improvement due to smoothing	270
9.2.2	Smoothing constant states	274
9.3	Fixed-lag smoothing	274
9.4	Fixed-interval smoothing	279
9.4.1	Forward-backward smoothing	280
9.4.2	RTS smoothing	286
9.5	Summary	294
	Problems	294

10 Additional topics in Kalman filtering	297
10.1 Verifying Kalman filter performance	298
10.2 Multiple-model estimation	301
10.3 Reduced-order Kalman filtering	305
10.3.1 Anderson's approach to reduced-order filtering	306
10.3.2 The reduced-order Schmidt–Kalman filter	309
10.4 Robust Kalman filtering	312
10.5 Delayed measurements and synchronization errors	317
10.5.1 A statistical derivation of the Kalman filter	318
10.5.2 Kalman filtering with delayed measurements	320
10.6 Summary	325
Problems	326

PART III THE H_∞ FILTER

11 The H_∞ filter	333
11.1 Introduction	334
11.1.1 An alternate form for the Kalman filter	334
11.1.2 Kalman filter limitations	336
11.2 Constrained optimization	337
11.2.1 Static constrained optimization	337
11.2.2 Inequality constraints	339
11.2.3 Dynamic constrained optimization	341
11.3 A game theory approach to H_∞ filtering	343
11.3.1 Stationarity with respect to x_0 and w_k	345
11.3.2 Stationarity with respect to \hat{x} and y	347
11.3.3 A comparison of the Kalman and H_∞ filters	354
11.3.4 Steady-state H_∞ filtering	354
11.3.5 The transfer function bound of the H_∞ filter	357
11.4 The continuous-time H_∞ filter	361
11.5 Transfer function approaches	365
11.6 Summary	367
Problems	369
12 Additional topics in H_∞ filtering	373
12.1 Mixed Kalman/ H_∞ filtering	374
12.2 Robust Kalman/ H_∞ filtering	377
12.3 Constrained H_∞ filtering	381
12.4 Summary	388
Problems	389

PART IV NONLINEAR FILTERS

13 Nonlinear Kalman filtering	395
13.1 The linearized Kalman filter	397
13.2 The extended Kalman filter	400
13.2.1 The continuous-time extended Kalman filter	400
13.2.2 The hybrid extended Kalman filter	403
13.2.3 The discrete-time extended Kalman filter	407
13.3 Higher-order approaches	410
13.3.1 The iterated extended Kalman filter	410
13.3.2 The second-order extended Kalman filter	413
13.3.3 Other approaches	420
13.4 Parameter estimation	422
13.5 Summary	425
Problems	426
14 The unscented Kalman filter	433
14.1 Means and covariances of nonlinear transformations	434
14.1.1 The mean of a nonlinear transformation	434
14.1.2 The covariance of a nonlinear transformation	437
14.2 Unscented transformations	441
14.2.1 Mean approximation	441
14.2.2 Covariance approximation	444
14.3 Unscented Kalman filtering	447
14.4 Other unscented transformations	452
14.4.1 General unscented transformations	452
14.4.2 The simplex unscented transformation	454
14.4.3 The spherical unscented transformation	455
14.5 Summary	457
Problems	458
15 The particle filter	461
15.1 Bayesian state estimation	462
15.2 Particle filtering	466
15.3 Implementation issues	469
15.3.1 Sample impoverishment	469
15.3.2 Particle filtering combined with other filters	477
15.4 Summary	480
Problems	481

Appendix A: Historical perspectives	485
Appendix B: Other books on Kalman filtering	489
Appendix C: State estimation and the meaning of life	493
References	501
Index	521