

Vadim I. Utkin

# Sliding Modes in Control and Optimization

With 24 Figures

Springer-Verlag

Berlin Heidelberg New York  
London Paris Tokyo  
HongKong Barcelona Budapest

# Contents

<b>Part I. Mathematical Tools</b> . . . . .	1
<b>Chapter 1 Scope of the Theory of Sliding Modes</b> . . . . .	1
1 Shaping the Problem . . . . .	1
2 Formalization of Sliding Mode Description . . . . .	7
3 Sliding Modes in Control Systems . . . . .	10
<b>Chapter 2 Mathematical Description of Motions on Discontinuity</b>	
<b>Boundaries</b> . . . . .	12
1 Regularization Problem . . . . .	12
2 Equivalent Control Method . . . . .	14
3 Regularization of Systems Linear with Respect to Control . . . . .	16
4 Physical Meaning of the Equivalent Control . . . . .	22
5 Stochastic Regularization . . . . .	25
<b>Chapter 3 The Uniqueness Problems</b> . . . . .	29
1 Examples of Discontinuous Systems with Ambiguous Sliding Equations . . . . .	29
1.1 Systems with Scalar Control . . . . .	30
1.2 Systems Nonlinear with Respect to Vector-Valued Control . . . . .	35
1.3 Example of Ambiguity in a System Linear with Respect to Control . . . . .	36
2 Minimal Convex Sets . . . . .	38
3 Ambiguity in Systems Linear with Respect to Control . . . . .	41
<b>Chapter 4 Stability of Sliding Modes</b> . . . . .	44
1 Problem Statement, Definitions, Necessary Conditions for Stability . . . . .	44
2 An Analog of Lyapunov's Theorem to Determine the Sliding Mode Domain . . . . .	46
3 Piecewise Smooth Lyapunov Functions . . . . .	50
4 Quadratic Forms Method . . . . .	55
5 Systems with a Vector-Valued Control Hierarchy . . . . .	59
6 The Finiteness of Lyapunov Functions in Discontinuous Dynamic Systems . . . . .	63

<b>Chapter 5 Singularly Perturbed Discontinuous Systems</b> . . . . .	66
1 Separation of Motions in Singularly Perturbed Systems . . . . .	66
2 Problem Statement for Systems with Discontinuous control . . . . .	68
3 Sliding Modes in Singularly Perturbed Discontinuous Control Systems . . . . .	70
 <b>Part II. Design</b> . . . . .	 75
 <b>Chapter 6 Decoupling in Systems with Discontinuous Controls</b> . . . . .	 76
1 Problem Statement . . . . .	76
2 Invariant Transformations . . . . .	78
3 Design Procedure . . . . .	80
4 Reduction of the Control System Equations to a Regular Form . . . . .	81
4.1 Single-Input Systems . . . . .	85
4.2 Multiple-Input Systems . . . . .	87
 <b>Chapter 7 Eigenvalue Allocation</b> . . . . .	 91
1 Controllability of Stationary Linear Systems . . . . .	91
2 Canonical Controllability Form . . . . .	94
3 Eigenvalue Allocation in Linear Systems. Stabilizability. . . . .	96
4 Design of Discontinuity Surfaces . . . . .	99
5 Stability of Sliding Modes . . . . .	104
6 Estimation of Convergence to Sliding Manifold . . . . .	108
 <b>Chapter 8 Systems with Scalar Control</b> . . . . .	 111
1 Design of Locally Stable Sliding Modes . . . . .	111
2 Conditions of Sliding Mode Stability “in the Large” . . . . .	115
3 Design Procedure: An Example . . . . .	121
4 Systems in the Canonical Form . . . . .	123
 <b>Chapter 9 Dynamic Optimization</b> . . . . .	 131
1 Problem Statement . . . . .	131
2 Observability, Detectability . . . . .	132
3 Optimal Control in Linear Systems with Quadratic Criterion . . . . .	135
4 Optimal Sliding Modes . . . . .	137
5 Parametric Optimization . . . . .	139
6 Optimization in Time-Varying Systems . . . . .	141
 <b>Chapter 10 Control of Linear Plants in the Presence of Disturbances</b> . . . . .	 145
1 Problem Statement . . . . .	145
2 Sliding Mode Invariance Conditions . . . . .	146

3 Combined Systems . . . . .	148
4 Invariant Systems Without Disturbance Measurements . . . . .	149
5 Eigenvalue Allocation in Invariant System with Non-measurable Disturbances . . . . .	151
<b>Chapter 11 Systems with High Gains and Discontinuous Controls . . . .</b>	<b>155</b>
1 Decoupled Motion Systems . . . . .	155
2 Linear Time-Invariant Systems . . . . .	157
3 Equivalent Control Method for the Study of Non-linear High-Gain Systems . . . . .	159
4 Concluding Remarks . . . . .	166
<b>Chapter 12 Control of Distributed-Parameter Plants . . . . .</b>	<b>169</b>
1 Systems with Mobile Control . . . . .	169
2 Design Based on the Lyapunov Method . . . . .	180
3 Modal Control . . . . .	184
4 Design of Distributed Control of Multi-Variable Heat Processes . . . .	186
<b>Chapter 13 Control Under Uncertainty Conditions . . . . .</b>	<b>189</b>
1 Design of Adaptive Systems with Reference Model . . . . .	189
2 Identification with Piecewise-Continuous Dynamic Models . . . . .	194
3 Method of Self-Optimization . . . . .	199
<b>Chapter 14 State Observation and Filtering . . . . .</b>	<b>206</b>
1 The Luenberger Observer . . . . .	206
2 Observer with Discontinuous Parameters . . . . .	207
3 Sliding Modes in Systems with Asymptotic Observers . . . . .	210
4 Quasi-Optimal Adaptive Filtering . . . . .	217
<b>Chapter 15 Sliding Modes in Problems of Mathematical Programming .</b>	<b>223</b>
1 Problem Statement . . . . .	223
2 Motion Equations and Necessary Existence Conditions for Sliding Mode . . . . .	226
3 Gradient Procedures for Piecewise Smooth Function . . . . .	227
4 Conditions for Penalty Function Existence. Convergence of Gradient Procedure . . . . .	230
5 Design of Piecewise Smooth Penalty Function . . . . .	232
6 Linearly Independent Constraints . . . . .	233
<b>Part III. Applications . . . . .</b>	<b>237</b>
<b>Chapter 16 Manipulator Control System . . . . .</b>	<b>239</b>

1 Model of Robot Arm . . . . .	240
2 Problem Statement . . . . .	240
3 Design of Control . . . . .	241
4 Design of Control System for a Two-joint Manipulator . . . . .	243
5 Manipulator Simulation . . . . .	246
6 Path Control . . . . .	248
7 Conclusions . . . . .	249
<b>Chapter 17 Sliding Modes in Control of Electric Motors . . . . .</b>	<b>250</b>
1 Problem Statement . . . . .	250
2 Control of d.c. Motor . . . . .	251
3 Control of Induction Motor . . . . .	255
4 Control of Synchronous Motor . . . . .	260
<b>Chapter 18 Examples . . . . .</b>	<b>265</b>
1 Electric Drives for Metal-cutting Machine Tools . . . . .	265
2 Vehicle Control . . . . .	269
3 Process Control . . . . .	271
4 Other Applications . . . . .	275
<b>References . . . . .</b>	<b>278</b>
<b>Subject Index . . . . .</b>	<b>285</b>