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Equilibrium Statistical Physics

Phases of Matter and Phase Transitions

With 82 Figures and 8 Tables

 Springer

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