AIMS Series on Applied Mathematics Volume 2

Introduction to the Mathematical Theory of Control

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With 102 figures and 107 exercises

American Institute of Mathematical Sciences

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