

ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

---

*Algorithmic Aspects of Graph Connectivity*

---

HIROSHI NAGAMOCHI

*Kyoto University*

TOSHIHIDE IBARAKI

*Kwansei Gakuin University*



**CAMBRIDGE**  
**UNIVERSITY PRESS**

# Contents

---

Preface	<i>page</i> ix
Notation	xi
<b>1 Introduction</b>	<b>1</b>
1.1 Preliminaries of Graph Theory	1
1.2 Algorithms and Complexities	13
1.3 Flows and Cuts	20
1.4 Computing Connectivities	34
1.5 Representations of Cut Structures	45
1.6 Connectivity by Trees	57
1.7 Tree Hypergraphs	60
<b>2 Maximum Adjacency Ordering and Forest Decompositions</b>	<b>65</b>
2.1 Spanning Subgraphs Preserving Connectivity	65
2.2 MA Ordering	73
2.3 3-Edge-Connected Components	86
2.4 2-Approximation Algorithms for Connectivity	100
2.5 Fast Maximum-Flow Algorithms	107
2.6 Testing Chordality	112
<b>3 Minimum Cuts</b>	<b>114</b>
3.1 Pendent Pairs in MA Orderings	114
3.2 A Minimum-Cut Algorithm	117
3.3 $s$ -Proper $k$ -Edge-Connected Spanning Subgraphs	119
3.4 A Hierarchical Structure of MA Orderings	123
3.5 Maximum Flows Between a Pendent Pair	127
3.6 A Generalization of Pendent Pairs	130
3.7 Practically Efficient Minimum-Cut Algorithms	131
<b>4 Cut Enumeration</b>	<b>137</b>
4.1 Enumerating All Cuts	137
4.2 Enumerating Small Cuts	140

4.3	Enumerating Minimum Cuts	145
4.4	Upper Bounds on the Number of Small Cuts	149
<b>5</b>	<b>Cactus Representations</b>	<b>153</b>
5.1	Canonical Forms of Cactus Representations	153
5.2	$(s, t)$ -Cactus Representations	171
5.3	Constructing Cactus Representations	180
<b>6</b>	<b>Extreme Vertex Sets</b>	<b>191</b>
6.1	Computing Extreme Vertex Sets in Graphs	192
6.2	Algorithm for Dynamic Edges Incident to a Specified Vertex	198
6.3	Optimal Contraction Ordering	200
6.4	Minimum $k$ -Subpartition Problem	207
<b>7</b>	<b>Edge Splitting</b>	<b>217</b>
7.1	Preliminaries	217
7.2	Edge Splitting in Weighted Graphs	220
7.3	Edge Splitting in Multigraphs	226
7.4	Other Splittings	232
7.5	Detachments	237
7.6	Applications of Splittings	240
<b>8</b>	<b>Connectivity Augmentation</b>	<b>246</b>
8.1	Increasing Edge-Connectivity by One	247
8.2	Star Augmentation	249
8.3	Augmenting Multigraphs	252
8.4	Augmenting Weighted Graphs	254
8.5	More on Augmentation	276
<b>9</b>	<b>Source Location Problems</b>	<b>282</b>
9.1	Source Location Problem Under Edge-Connectivity Requirements	283
9.2	Source Location Problem Under Vertex-Connectivity Requirements	295
<b>10</b>	<b>Submodular and Posimodular Set Functions</b>	<b>304</b>
10.1	Set Functions	304
10.2	Minimizing Submodular and Posimodular Functions	306
10.3	Extreme Subsets in Submodular and Posimodular Systems	315
10.4	Optimization Problems over Submodular and Posimodular Systems	320
10.5	Extreme Points of Base Polyhedron	336
10.6	Minimum Transversal in Set Systems	342
	Bibliography	357
	Index	371