Aggregation Functions

MICHEL GRABISCH

University of Panthéon-Sorbonne Paris, France

JEAN-LUC MARICHAL

University of Luxembourg
Luxembourg

RADKO MESIAR

Slovak University of Technology Bratislava, Slovakia

ENDRE PAP

University of Novi Sad Novi Sad, Serbia



Contents

Li	st of fig	gures	page x	
List of tables				
Pr	eface		xiii	
1	Introduction			
	1.1	Main motivations and scope	1	
	1.2	Basic definitions and examples	2	
	1.3	Conventional notation	9	
2	Properties for aggregation			
	2.1	Introduction	11	
	2.2	Elementary mathematical properties	12	
	2.3	Grouping-based properties	31	
	2.4	Invariance properties	41	
	2.5	Further properties	49	
3	Conjunctive and disjunctive aggregation functions			
	3.1	Preliminaries and general notes	56	
	3.2	Generated conjunctive aggregation functions	59	
	3.3	Triangular norms and related conjunctive aggregation functions	64	
	3.4	Copulas and quasi-copulas	88	
	3.5	Disjunctive aggregation functions	100	
	3.6	Uninorms	106	
	3.7	Nullnorms	115	
	3.8	More aggregation functions related to t-norms	119	
	3.9	Restricted distributivity	123	
4	Means and averages			
	4.1	Introduction and definitions	130	
	4.2	Quasi-arithmetic means	132	

viii Contents

	4.3	Generalizations of quasi-arithmetic means	139	
	4.4	Associative means	161	
	4.5	Means constructed from a mean value property	163	
	4.6	Constructing means	166	
	4.7	Further extended means	168	
_			1.71	
5		egation functions based on nonadditive integrals	171	
	5.1	Introduction	171	
	5.2	Set functions, capacities, and games	172	
	5.3	Some linear transformations of set functions	177	
	5.4	The Choquet integral	181	
	5.5	The Sugeno integral	207	
	5.6	Other integrals	227	
6	Construction methods			
	6.1	Introduction	234	
	6.2	Transformed aggregation functions	234	
	6.3	Composed aggregation	242	
	6.4	Weighted aggregation functions	247	
	6.5	Some other aggregation-based construction methods	252	
	6.6	Aggregation functions based on minimal dissimilarity	257	
	6.7	Ordinal sums of aggregation functions	261	
	6.8	Extensions to aggregation functions	266	
7	Aggregation on specific scale types			
	7.1	Introduction	272	
	7.2	Ratio scales	273	
	7.3	Difference scales	280	
	7.4	Interval scales	284	
	7.5	Log-ratio scales	289	
8	Aggr	egation on ordinal scales	292	
Ü	8.1	Introduction	292	
	8.2	Order invariant subsets	293	
	8.3	Lattice polynomial functions and some of their properties	296	
	8.4	Ordinal scale invariant functions	300	
	8.5	Comparison meaningful functions on a single ordinal scale	304	
	8.6	Comparison meaningful functions on independent ordinal scales	308	
	8.7	Aggregation on finite chains by chain independent functions	310	
	0.7	1-06-76-1011 on time channe of chain independent functions	510	
9	-	egation on bipolar scales	317	
	9.1	Introduction	317	
	9.2	Associative bipolar operators	319	

		Contents	ix
ç	9.3	Minimum and maximum on symmetrized linearly ordered sets	325
ç	9.4	Separable aggregation functions	332
9	9.5	Integral-based aggregation functions	334
10 I	Behav	vioral analysis of aggregation functions	348
1	10.1	Introduction	348
- 1	10.2	Expected values and distribution functions	348
. 1	10.3	Importance indices	361
- 1	10.4	Interaction indices	367
1	10.5	Maximum improving index	370
1	10.6	Tolerance indices	372
1	10.7	Measures of arguments contribution and involvement	378
11 I	dent	ification of aggregation functions	382
1	11.1	Introduction	382
. 1	11.2	General formulation	383
1	11.3	The case of parametrized families of aggregation functions	386
. 1	11.4	The case of generated aggregation functions	388
1	11.5	The case of integral-based aggregation functions	391
, , 1	11.6	Available software	396
App	endix	A: Aggregation of infinitely many arguments	397
A	4.1	Introduction	397
A	4.2	Infinitary aggregation functions on sequences	397
A	4.3	General aggregation of infinite number of inputs	405
App	endix	x B: Examples and applications	410
E	3.1	Main domains of applications	410
F	3.2	A specific application: mixture of uncertainty measures	414
List	of syr	nbols	420
Refe	References		
Inde.			454