## **HANDBOOK** OF **SEQUENTIAL ANALYSIS**

edited by

B. K. GHOSH P. K. SEN

Bethlehem, Pennsylvania

Lehigh University University of North Carolina Chapel Hill, North Carolina

## Contents

Prefa Cont		tors	$egin{array}{c} iii \ xiii \end{array}$
1.		Brief History of Sequential Analysis  K. Ghosh	1
	1.	Introduction	1
	2.	The conception: 1657–1943	5
	3.	The birth and childhood: 1943–1950	7
	4.	From adolescence to adulthood: 1951–1990	10
		References	14
2.		vo-Stage and Multistage Procedures outir Kishore Chatterjee	21
	1.	Introduction	21
	2.	Nonexistence of fixed sample-size procedures whose performance is free from nuisance parameters	22
	3.	Two-stage procedures for inferring about the univariate normal mean	24
	4.	Two-stage procedures for the replicable linear regression Aitken setup	29
	5.	Two-stage procedures for inferring about the multinormal mean vector	31

	6. Two-stage procedures for the general linear regression setup: nonstochastic and stochastic predictors	36
	7. Procedures of Stein's type for other problems and some related studies	39
	8. A shortcoming of procedures of Stein's type: remedies References	41 44
3.	The Sequential Probability Ratio Test Bennett Eisenberg and B. K. Ghosh	47
	1. Introduction	47
	2. Sequences of likelihood ratios	48
	3. SPRTs in the iid case	52
	4. SPRTs for one-sided hypotheses	53
	5. Average sample size and optimality of the SPRT	56
	6. Relative efficiency of the SPRT	58
	7. Multiparameter families and the invariant SPRT	60
	References	64
4.	Stopping Times: Termination, Moments, Distribution Robert A. Wijsman	67
	1. Introduction and general tools	67
	2. Tests with straight-line stopping bounds	75
	3. Tests with curved stopping bounds	94
	4. Multiple-decision procedures	107
	5. Confidence intervals and sets	108
	References	114
5.	Asymptotic Optimality of Generalized Sequential Likelihood Ratio Tests in Some Classical Sequential Testing Problems Tze-Leung Lai	121
	1. Introduction	121
	2. The Kiefer-Weiss problem and 2-SPRT	124
	3. Asymptotic theories of Chernoff and Schwarz for Bayes sequential tests of one-sided hypotheses	128
	4. Generalized sequential likelihood ratio tests and a unified asymptotic theory for Bayes sequential tests of one-sided hypotheses	131
	5. Asymptotic optimality of an invariant SPRT and of SPRTs based on dependent observations	139
	References	142
6.	The Role of Renewal Theory in Sequential Analysis  Michael Woodroofe	145
	1. Introduction	145
	2. Random walks and the renewal theorem	146
	3. The sequential probability ratio test	150
	4. Nonlinear renewal theorems	152
	5. Integrated versions	157

Con	tents	vii
	<ul><li>6. Sequential testing</li><li>7. Sequential estimation References</li></ul>	160 163 166
7.	Repeated Significance Tests in Frequency and Time Domains P. K. Sen	169
	<ol> <li>Introduction</li> <li>RST, GST, TSPRT, and TSLRT: frequency domain</li> <li>RST and PCS in time domain References</li> </ol>	169 171 186 196
8.	Multivariate Problems Bimal K. Sinha	199
	<ol> <li>Introduction</li> <li>Inference about μ from N(μ, Σ)</li> <li>Inference about regression parameters References</li> </ol>	199 200 215 225
9.	Multihypothesis Problems Bennett Eisenberg	229
	<ol> <li>Introduction</li> <li>Criteria for the evaluation of multihypothesis tests</li> <li>Sequential multihypothesis tests</li> <li>Tests based on rejection of the alternatives         References     </li> </ol>	229 230 232 240 243
10.	Parametric Sequential Point Estimation N. Mukhopadhyay	245
	<ol> <li>Introduction</li> <li>Cramér-Rao bound and other results</li> <li>Estimating the mean of a normal population having unknown variance</li> </ol>	245 246 247
	<ol> <li>Estimating the percentile of a normal population</li> <li>Multivariate problems</li> <li>Comparing means of several normal populations</li> <li>Other specific problems and comments         References     </li> </ol>	250 253 257 259 264
11.	Confidence Sets and Intervals $Jana\ Jure\check{c}kov\acute{a}\ ackslash$	269
	<ol> <li>Introduction</li> <li>Bounded length sequential confidence intervals for the location</li> </ol>	269 271
	parameter 3. Sequential confidence intervals for regression parameters References	278 279

viii Contents

12.	Group Sequential Tests and Repeated Confidence Intervals Christopher Jennison and Bruce Turnbull			
	<ol> <li>Introduction and historical review</li> <li>Group sequential tests of a null hypothesis against a two-sided alternative</li> </ol>	283 285		
	3. Group sequential tests of a null hypothesis against a one-sided alternative	290		
	4. Unequal group sizes	294		
	5. Analysis following a group sequential test	299		
	6. Repeated confidence intervals References	304 308		
13.	Sequential Methods for Finite Populations	313		
	Bikas Kumar Sinha			
	1. Introduction	313		
	2. Sequential sampling designs	314		
	3. Bounded risk estimation of a finite population mean: comparison of strategies based on balanced sampling schemes	319		
	4. Bayes sequential sampling designs	325		
	References	328		
14.	Nonparametric Methods in Sequential Analysis P. K. Sen	331		
	1. Introduction	331		
	2. Nonparametric sequential tests	333		
	3. Minimum risk sequential nonparametric estimation	344		
	4. Nonparametric sequential confidence sets	352		
	5. Nonparametrics in change-point models	357		
	References	360		
15.	Sequential Ranking and Selection Procedures Shanti S. Gupta and S. Panchapakesan	363		
	1. Introduction	363		
	2. Some general aspects of sequential selection procedures	364		
	3. Selection from normal populations: IZ approach	366		
	4. Selection from Bernoulli populations: IZ approach	370		
	5. Selection from normal populations: SS approach	371		
	6. Selection from Bernoulli populations: SS approach	374		
	7. Selection from exponential family	375		
	8. Concluding remarks	377		
	Summary	377		
	References	377		
16.	Secretary Problems	381		
	Stephen M. Samuels			
	1. Introduction	381		
	2. The "classical" best-choice problem	382		
	3. Googol	383		

Contents	ix
----------	----

	4. A general class of problems	386
	5. An infinite model	394
	6. Multiple criteria, multiple choices, multiple choosers	400
	References	402
17.	Sequential Decision Theory Norbert J. Schmitz	407
	1. Introduction	407
	2. The basic model	407
	3. Sequential decision procedures	411
	4. Risk function, admissibility	412
	5. Sufficiency and transitivity	416
	6. The II-minimax principle	419
	7. Sequential Bayesian analysis	420
	8. Sequential minimax analysis	423
	9. Sequential sampling plans	426
	References	427
18.	Asymptotically Pointwise Optimal Stopping Rules  Malay Ghosh and Robert M. Hoekstra	429
	1. Introduction	429
	2. Asymptotic pointwise optimality and asymptotic optimality	430
	3. Asymptotic nondeficiency	433
	4. Set estimation	436
	References	438
19.	Hierarchical and Empirical Bayes Sequential Estimation  Malay Ghosh	441
	1. Introduction	441
	2. Estimation of the multivariate normal mean	444
	3. Estimation of the regression coefficient	451
	References	457
20.	Sequentially Adaptive Nonparametric Procedures  Marie Hušková	459
		459
	1. Introduction 2. General features	461
	3. Density estimators	464
	4. Estimators of the optimal score-generating functions connected	468
	with $R$ -, $L$ - and $M$ -estimators	100
	5. Concluding remarks	473
	References	474
21.	Sequential Design and Allocation Rules A. Basu, A. Bose, and J. K. Ghosh	475
	1. Introduction	475
	2. Historical background and early results	476
	3. The <i>n</i> -armed bandit problems	477

x	/	Contents
•		

	4. 5.	Allocation and analysis of clinical trials Other works References	479 487 488
22.		chastic Approximation $yid$ $Ruppert$	503
	1. 2. 3. 4. 5. 6.	Introduction The Robbins-Monro process The Kiefer-Wolfowitz process Multivariate SA and Monte Carlo optimization Stopping rules and confidence intervals Stochastic control References	503 505 516 518 524 525 527
23.		tection and Change-Point Problems  Zacks	531
	1. 2.	Introduction Bayes sequential procedures for detecting change points: the distributions before and after the change are known	531 532
	3. 4. 5.	The distributions before and after the change are not known CUSUM procedures for sequential detection Sequential detection by process tracking References	539 544 553 560
24.	-	ality Control bert John Rowlands and G. Barrie Wetherill	563
	1. 2. 3. 4.	Introduction Control charts for independent observations Two-sided charts for independent observations Autocorrelated observations References	563 564 571 574 578
25.		quential Methods in Reliability and Life Testing it $P. Basu$	581
	1. 2. 3. 4. 5.	Introduction Exponential distribution Other parametric models Sequential probability ratio test for availability Estimation of reliability Two-stage and repeated significance test procedures for distributions with aging concepts References	581 583 585 587 587 590
26.		quential Methods in Clinical Trials nn Whitehead	593
	1. 2. 3.	Introduction A review of the use of sequential methods in clinical research A study of immunosuppressives in bone marrow transplantation	593 594 600

Contents		xi		
	4.	Future developments References	606 608	
27.		me-Sequential Estimation seph C. Gardiner and V. Susarla	613	
	1.	Introduction and motivation	613	
	2.	Time-sequential estimation of the exponential mean	617	
	3.	Time-sequential estimation of the exponential mean under random censorship	619	
	4.	Time-sequential estimation in general parametric models under censorship	625	
	5.	Concluding remarks	629	
		References	630	
Inde	v		633	

.