

# Free-Surface Flows in Shape Optimization of Extrusion Dies

Von der Fakultät für Maschinenwesen  
der Rheinisch-Westfälischen Technischen Hochschule Aachen  
zur Erlangung des akademischen Grades einer  
Doktorin der Ingenieurwissenschaften  
genehmigte Dissertation

vorgelegt von  
Stefanie Nicole Elgeti, geb. Barth

Berichter: Univ. Prof. Marek Behr, Ph.D.  
Univ. Prof. Dr.-Ing. Dr.-Ing. E.h. Walter Michaeli

Tag der mündlichen Prüfung: 15. Juni 2011

---

# Contents

---

<b>List of figures</b>	<b>x</b>
<b>List of tables</b>	<b>xi</b>
<b>Notation</b>	<b>xii</b>
<b>Kurzfassung</b>	<b>xv</b>
<b>1 Introduction</b>	<b>1</b>
1.1 The Manufacturing Process Extrusion . . . . .	1
1.2 Computational Design of Extrusion Dies . . . . .	4
1.3 Overview . . . . .	5
<b>2 Governing Equations</b>	<b>7</b>
2.1 Generalized Newtonian Constitutive Equations . . . . .	8
2.1.1 Carreau Model . . . . .	9
2.2 Constitutive Equations Representing Viscoelasticity . . . . .	10
2.2.1 Maxwell Model . . . . .	11
2.2.2 Oldroyd-B Model . . . . .	12
2.2.3 Giesekus Model . . . . .	12
2.3 Characteristic Values . . . . .	13
2.3.1 Deborah Number . . . . .	13
2.3.2 Weissenberg Number . . . . .	13
<b>3 XNS Shape Optimization</b>	<b>15</b>
3.1 The Flow Solver XNS . . . . .	15
3.1.1 The Finite Element Method . . . . .	15
3.1.2 Free-Surface Flows . . . . .	19
3.1.3 Space-Time Finite Element Methods . . . . .	20
3.1.4 Iterative Solution Techniques . . . . .	21

3.2	Objective Function	25
3.2.1	Objective Function Measuring the Homogeneity of the Velocity Distribution	26
3.2.2	Objective Function Measuring Homogeneous Die Swell	28
3.3	Geometry Representation	30
3.3.1	Non-Uniform Rational B-Splines (NURBS)	31
3.3.2	Shape Update: $\alpha \rightarrow \gamma$	32
3.3.3	Mesh Update: $\gamma \rightarrow x$	34
3.4	Optimizer	34
3.4.1	DONLP2	36
3.4.2	BOBYQA	37
3.5	Materials	37
3.5.1	Acrylonitrile butadiene styrene (ABS)	38
3.5.2	Polyvinyl chloride (PVC)	38
<b>4</b>	<b>Optimization Regarding a Homogeneous Velocity Distribution</b>	<b>39</b>
4.1	Variational Form of the Stokes Equations	39
4.2	Geometry Deformation	41
4.3	Behavior of the Objective Function	42
4.3.1	Test Geometry	42
4.3.2	Behavior of the Objective Function for Different Numbers of Subsections	45
4.3.3	Behavior of the Objective Function for Different Materials	47
4.4	Application to Industrially Relevant Extrusion Dies	50
4.4.1	Initial Geometry	50
4.4.2	L-Profile	51
4.4.3	L-Profile with Full Design Concept	59
4.4.4	Floor Skirting Profile	63
4.4.5	Meteor Profile	65
4.5	Good Parameterizability	68
<b>5</b>	<b>Optimization Regarding Homogeneous Die Swell</b>	<b>73</b>
5.1	Implementation of the Viscoelastic Models in XNS	73
5.1.1	Variational Form of the Stokes Equations in the Velocity-Pressure-Stress Formulation	74
5.1.2	Boundary Conditions	76
5.1.3	Influence on the Velocity Distribution	77
5.2	Die Swell	78
5.2.1	Representation of the Free Surface	78
5.2.2	Comparison of Die Swell Using the Oldroyd-B and the Giesekus Model (in 2D)	80
5.2.3	Complications in Three Space Dimensions	88
5.2.4	Mesh Reduction	89
5.2.5	NURBS Approximation of the Free Surface	90
5.2.6	Features which Fail with Standard Mesh Deformation	93
5.3	Optimization Regarding Die Swell	94

5.3.1	Two Space Dimensions . . . . .	94
5.3.2	Three Space Dimensions . . . . .	96
<b>6</b>	<b>Conclusion</b>	<b>99</b>
6.1	Future Research Directions . . . . .	101
	<b>Bibliography</b>	<b>102</b>