

Statistical Signal Processing of Complex-Valued Data

The Theory of Improper and
Noncircular Signals

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