Random Coefficient Models

NICHOLAS T. LONGFORD

Educational Testing Service
Princeton, New Jersey

CLARENDON PRESS · OXFORD

Contents

1	**Introduction**		1
	1.1	Clustered observations	1
		1.1.1 Subjects and variables	4
	1.2	Matrices	4
		1.2.1 Matrix operations and properties	5
		1.2.2 Matrix differentiation	7
	1.3	Normal distribution	8
		1.3.1 Conditional distributions	9
	1.4	Ordinary regression	10
		1.4.1 Analysis of variance	11
	1.5	Examples	12
		1.5.1 Commuting times	12
		1.5.2 Blood pressures	13
	1.6	Fixed versus random	15
	1.7	Shrinkage estimation of cluster means	16
	1.8	Sources of variation	18
	1.9	Maximum likelihood	19
		1.9.1 Newton–Raphson method	20
	1.10	The exponential family of distributions	22
	1.11	Bibliographical notes	23
2	**Analysis of covariance with random effects**		26
	2.1	Models	26
		2.1.1 The log-likelihood	28
	2.2	Examples	30
		2.2.1 Financial ratios	30
		2.2.2 Rat weights	32
		2.2.3 Monitoring pregnancy	34
		2.2.4 House prices	36
		2.2.5 Validity of an educational test	38
	2.3	Newton–Raphson and Fisher scoring algorithms	40
		2.3.1 Some technical details	43
	2.4	Generalized least squares	46
	2.5	EM algorithm	47
		2.5.1 The E-step	48
		2.5.2 The M-step	49

	2.6	Restricted maximum likelihood	49
	2.7	Balanced design	52
	2.8	The price of ignoring clustering	53
	2.9	Cluster size and information	56
	2.10	Residuals. Model checking	60
		2.10.1 Shrinkage estimators	61
	2.11	Measures of quality of the model fit	62
	2.12	Bibliographical notes	62
3	**Examples. Random-effects models**		64
	3.1	Financial ratios	64
	3.2	Rat weights	74
	3.3	Monitoring pregnancy	78
	3.4	House prices	83
	3.5	GRE validity study	86
		3.5.1 Estimation based on the within-department means	91
4	**Random regression coefficients**		94
	4.1	Models	94
	4.2	Invariance and linear transformations	97
		4.2.1 Invariance in ordinary regression	97
		4.2.2 Random coefficients and invariance	98
	4.3	Patterns of variation	100
		4.3.1 Categorical variables and variation	104
	4.4	Maximum likelihood estimation	106
		4.4.1 Constrained maximization	110
		4.4.2 Confounding in the variation part	111
	4.5	Longitudinal analysis	112
	4.6	Multivariate regression	116
		4.6.1 Multivariate and longitudinal data	118
	4.7	REML estimation	123
	4.8	Model checking	124
	4.9	General patterns of dependence	124
		4.9.1 General form of elementary-level variance	126
	4.10	Bibliographical notes	127
5	**Examples using random coefficient models**		128
	5.1	Financial ratios	128
	5.2	Rat weights	136
	5.3	Pregnancy monitoring	137
	5.4	House prices	141
	5.5	GRE validity study	145

6 Multiple levels of nesting 156
6.1 Models 156
6.1.1 Level-wise equations 159
6.2 Estimation 160
6.2.1 Organizing computations 165
6.3 Model choice 167
6.4 Restricted maximum likelihood 169
6.5 Model diagnostics 170
6.6 Likelihood ratio testing 171
6.6.1 Independent data? 172
6.6.2 An irrelevant variable 173
6.7 More than three levels of nesting 173
6.8 Hearing loss data 175
6.9 Model based estimation in surveys 184
6.9.1 A model based alternative 187
6.10 Bibliographical notes 189

7 Factor analysis and structural equations 190
7.1 Introduction 190
7.2 Factor analysis 190
7.3 Maximum likelihood estimation 192
7.3.1 Exploratory mode 194
7.4 Two-level factor analysis 195
7.4.1 Maximum likelihood estimation 197
7.4.2 Constrained maximization 201
7.5 Restricted maximum likelihood 202
7.6 Saturated model (starting solution) 203
7.7 Inference about $\bar{\mathbf{y}}$ 204
7.8 Measuremement error models 205
7.9 A two-level measurement error model 209
7.10 General covariance structures 212
7.11 Example. Second International Mathematics Study 212
7.12 Hearing loss data 215
7.13 Bibliographical notes 218

8 GLM with random coefficients 219
8.1 Introduction 219
8.2 Models for independent observations 219
8.2.1 Generalized least squares 223
8.3 Quasilikelihood 223
8.3.1 Extended quasilikelihood 224
8.4 Models for clustered observations 225
8.5 Maximum likelihood estimation 227

8.5.1 Direct maximization 228
8.5.2 Approximation of the integrand 230
8.5.3 Approximate Fisher scoring algorithm 231
8.6 Information about variation 235
8.7 Restricted maximum likelihood 236
8.8 Example. Interviewer variability 237
8.9 Death rates of Medicare patients 244
8.10 Bibliographical notes 248

9 Appendix. Asymptotic theory 250
9.1 Limited variance of the scoring function 253
9.2 Asymptotic normality of the scoring vector 254
9.3 Consistency of MLE 254
9.4 Asymptotic normality of MLE 255

References 257

Index 266