Modern Mathematical Methods of Optimization

edited by Karl-Heinz Elster

Akademie Verlag

Contents

Introduction 15

1 *Modern numerical methods and software in optimization* 23

1.1 The linearization method in constrained optimization 23

1.1.1 Introduction 23

1.1.2 The essence of the method, its formulation and convergence conditions 24

1.1.3 Exact penalty functions 27

1.1.4 Selection of N and constructivity of convergence conditions . . 29

1.1.5 Local analysis 33

1.1.6 A linearization method with accelerated convergence 37

1.1.7 Some questions of implementation 39

1.2 Superlinearly convergent methods of nonlinear programming 43

1.2.1 Introduction 43

1.2.2 Local theory 43

1.2.3 Global theory 47

1.3 Optimization problems in the presence of noise 53

1.3.1 Sources and types of noise 53

1.3.2 Behaviour of standard methods in the presence of noise 54

1.3.3 Convergence of methods under random noise 55

1.3.4 Optimal methods 56

1.3.5 Generalizations 57

1.3.6 Implementable algorithms 59

1.3.7 Constrained minimization 60

1.3.8 Conclusion 61
1.4 Software for mathematical programming problems 61
References 65

2 Optimal methods of convex programming and polynomial methods of linear programming 75

2.1 Local methods of convex programming. Optimal methods 75
2.1.1 Local methods 75
2.1.2 Complexity of convex problems 76
2.1.3 Cutting plane methods 77
2.1.4 Method of centres 78
2.1.5 Method of circumscribed ellipsoids 79
2.1.6 Method of inscribed ellipsoids 79
2.1.7 Constrained problems 81
2.2 Polynomial algorithms in linear programming 82
2.2.1 Algebraic statement of the problem 82
2.2.2 Bit posing. Method of ellipsoids 83
2.2.3 Karmarkar's method 84
2.2.4 The projection method 86
2.2.5 The methods of Renegar and Vaidya 89
2.2.6 Dual algorithms 91
2.3 Optimal methods for the solution of large-scale convex programming problems 97
2.3.1 Smooth spaces 98
2.3.2 General convex problems 99
2.3.3 Smooth convex problems 101
2.3.4 Problems with regular minimum 107
References 111
Additional References 114

3 Decomposition of optimization problems 116

3.1 Decomposition approach in optimization 116
3.1.1 Typical approaches 117

3.1.2 Generalizations 121
3.2 Convex problems with bordering structure 123
3.2.1 Statement of the problem 123
3.2.2 Primal decomposition with allocation of centralized resources . 125
3.2.3 Simultaneous application of primal and dual decomposition . . 128
3.3 A general approach to the construction of decomposition (block) methods of convex programming 132
3.3.1 Statement of the problem 132
3.3.2 Description of the method 134
3.3.3 Discussion of the method 136
3.3.4 Decomposition method for the analysis of monotonic mappings 138
References 142

4 Nonsmooth functions **146**
4.1 Nonsmooth analysis and directional derivatives 146
4.1.1 Introduction 146
4.1.2 Approximation of functions 147
4.1.3 Dini derivatives 150
4.1.4 Clarke derivatives 152
4.1.5 Approximation of sets 155
4.1.6 Clarke's cone 158
4.2 Quasidifferentiable functions 159
4.3 Lexicographically smooth functions 164
4.3.1 The class of lexicographically smooth functions 164
4.3.2 Lexicographic derivatives and their properties 165
4.3.3 Fundamental theorem of lexicographic differential calculus . . . 168
4.3.4 Examples of lexicographic derivatives 170
4.3.5 Some theorems from analysis 173
References 174

5 Improper problems of mathematical programming **178**
5.1 Introduction 178
5.2 Duality 180

5.2.1 Realization of duality for improper linear programming problems 181
5.2.2 Duality for improper convex programming problems of the first kind . . . 187
5.2.3 On duality of improper problems in infinite-dimensional spaces 188
5.3 Infinite-dimensional linear programming problems with a duality gap . 191
5.3.1 Approximation of an infinite-dimensional LP problem by cones 191
5.3.2 Infinite linear programming problems over R_∞ . . . 193
5.4 Correction of systems of linear equations and inequalities using additional information . . . 195
5.5 Correction of improper minimax problems . . . 198
5.5.1 Solvability sets . . . 198
5.5.2 Correction according to a convex criterion . . . 199
5.5.3 Correction with respect to a concave-convex criterion . . . 201
5.6 Regularization of improper linear problems . . . 203
5.7 Correction of improper convex programming problems by means of augmented Lagrangians . . . 206
5.7.1 Correction on the basis of a modification of the Lagrangian with respect to dual variables . . . 206
5.7.2 Correction on the basis of a symmetric modification of the Lagrangian . . . 209
References . . . 211

6 Optimization in order scales 212
6.1 Introduction . . . 212
6.2 Conceptions and mechanisms of choice . . . 213
6.3 Choice functions . . . 214
6.3.1 Choice and choice function . . . 214
6.3.2 Partial and complete choice functions . . . 214
6.3.3 Characteristic features of a choice function . . . 215
6.4 Binary relations . . . 216
6.4.1 Measurement of the quality of a solution . . . 216
6.4.2 Classes of binary relations . . . 217
6.4.3 Indicators . . . 219
6.5 Binary relations and choice functions . . . 220

6.5.1 Classes of choice functions 220
6.5.2 Characterizing properties of certain choice functions 222
6.5.3 Decomposition of choice functions 223
6.6 Optimization with respect to a binary relation 224
6.6.1 Optimal elements 224
6.6.2 Comparison of variants 225
6.6.3 Correcting choice functions 227
6.7 Generalized mathematical programming 227
6.7.1 The development of models 227
6.7.2 Mathematical programming in order scales 228
6.7.3 Solution methods 229
6.7.4 Concluding remarks 231
References 231

7 Multiobjective optimization 233
7.1 Duality in multiobjective optimization 233
7.1.1 Strong and weak duality 234
7.1.2 Construction of dual problems 235
7.1.3 Duality for multiobjective location problems 239
7.1.4 Duality in integer multiobjective programming 240
7.2 Duality of choice in vector optimization problems 242
7.2.1 Primal and dual choice functions 242
7.2.2 Dualization of preferences and alternatives 243
7.2.3 Duality and Pareto optimality 244
7.2.4 Pareto optima and polar correspondences 246
7.2.5 Dual generalized criteria 248
7.2.6 An application 250
7.2.7 Duality for asymmetric budget sets 251
7.3 Multicriteria optimization problems involving importance-ordered criteria 254
7.3.1 Basic definitions 254
7.3.2 Main kinds of importance 257
7.3.3 Problems involving homogeneous importance-ordered criteria . 260

7.3.4 Convex problems with homogeneous criteria ordered with respect to importance 262
7.3.5 Multicriteria problems with inhomogeneous criteria ordered with respect to importance 264
7.4 Efficiency estimation of decision rules in discrete multiobjective problems 267
7.4.1 Construction of solutions 268
7.4.2 Efficiency indicators of decision rules in multicriteria problems 270
7.4.3 Efficiency estimates for twocriterial problems 273
7.4.4 Some efficiency estimates for the general case 275
7.5 On the dialogue procedures of decision making in practical multicriteria models of economy 277
7.5.1 Statement of the problem and optimization procedures 277
7.5.2 An example of applying the dialogue systems described 279
7.5.3 A program package of multicriteria optimization 280
7.5.4 A method of interactive multicriteria optimization 281
References 282

8 Discrete optimization **288**
8.1 Mathematical models and some applied problems 288
8.1.1 The general problem of ILP 289
8.1.2 The multidimensional knapsack problem 289
8.1.3 The onedimensional knapsack problem 290
8.1.4 The problems of packing, partitioning and covering a set 290
8.1.5 The travelling salesman problem 291
8.1.6 The fixed-charge problem 292
8.1.7 Consideration of problem specifics 293
8.1.8 The quadratic assignment problem 294
8.1.9 Problems of projecting mining enterprises 295
8.1.10 Selection of optimal technologies 295
8.1.11 Other models and applications 296
8.2 *Methods of discrete optimization. Complexity of discrete problems* . . 297
8.2.1 General survey 297
8.2.2 Behaviour of methods 298

8.2.3 Elements of complexity theory 299
8.2.4 Complexity of discrete optimization problems 301
8.2.5 A way out of the situation 302
8.3 Effective approximation methods of discrete optimization 304
8.3.1 Polynomial approximation schemes 304
8.3.2 Polynomial approximately feasible algorithms 308
8.4 The knapsack problem and its extensions 310
8.4.1 The knapsack problem 310
8.4.2 The multiple-choice knapsack problem 313
8.4.3 The multistep knapsack problem 315
8.4.4 Linear generalizations 316
8.4.5 Nonlinear generalizations 316
8.4.6 A special problem 318
8.5 Dual approach in integer programming 318
8.5.1 Duality in ILP 318
8.5.2 Generalized labelling methods 323
References 326

9 Some problems of mathematical programming in infinite-dimensional spaces 337

9.1 Duality assertions and optimality conditions of general fractional optimization problems 337
9.1.1 Introduction 337
9.1.2 Conjugate functions 338
9.1.3 Duality assertions and optimality conditions 340
9.1.4 Comparison of conjugation concepts 345
9.2 Optimal control and duality in control problems 347
9.2.1 Fundamental problems of optimal control 347
9.2.2 Variational problems 349
9.2.3 Pontryagin's maximum principle 350
9.2.4 Dual problems for optimal control problems 351
9.3 Numerical analysis and solution methods for semi-infinite programming problems 353

9.3.1 Introduction 353
9.3.2 Theoretical background of numerical methods 355
9.3.3 Numerical methods for semi-infinite problems 359
9.4 Proximity-space methods in optimization with constraints 362
9.4.1 Introduction 362
9.4.2 Topological background 363
9.4.3 Optimization problems treated with tolerance 365
9.4.4 Compactification of optimization problems 368
References 369

10 Optimization and mathematical economics **375**
10.1 Price regulation in the presence of queues and quantity rationing . . . 375
10.1.1 Demand and fixed price equilibrium in the presence of queues . 376
10.1.2 Demand and fixed price equilibrium under quantity rationing . 379
10.1.3 Price regulation 381
10.1.4 Concluding remarks 383
10.2 Processes of finding equilibrated states 383
10.2.1 Equilibrated states 384
10.2.2 First method of searching equilibrated states: movement in the space of weighted coefficients 388
10.2.3 Second approach to finding equilibrated states: movement in the original space 390
10.3 Local estimates and the selection problem 395
10.3.1 Selection functions 396
10.3.2 The problem of multicriteria optimization 397
10.3.3 Regularization of contradictory requirements 398
10.3.4 The problem of quasilinear programming 400
10.4 Computation of equilibria for a class of piecewise linear models by a sequence of linear programs 401
References 407

Index **411**